Urban Economics and Analysis

Slides B

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Outline

Introduction

As we already have seen, transport costs play a very important role in (traditional) Urban Economics. Now we are going to consider congestion.

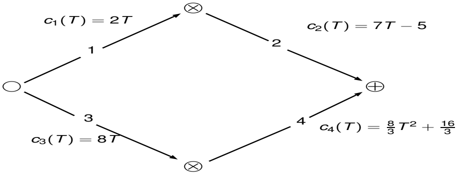
The aim of Slides B is to better understand the congestion model in Chapter 5 of the text book of Brueckner (see Slides A). In fact the analysis in this chapter is in some sense a game theoretical one, but the game theory is hidden. In addition the model makes some heroic assumptions.

The nice thing is that, as we shall see, it is very well possible to make the involved game theory explicit. In doing so the model becomes also more realistic.

Below You can find a quick and efficient route for understanding the very very basics of congestion games.

Simple traffic network

Let us start with a very simple motivating example by considering the following traffic network:



Simple traffic network (ctd.)

The intended interpretation is as follows.

- Each morning n commuters want to go from node (i.e. place) $\bigcirc$ to node $\bigoplus$.
- There are 4 roads: 1, 2, 3, 4. The configuration of these roads makes that there are two possible routes for commuting: roads 1–2 (route 1) and roads 3–4 (route 2).
- $c_j(T)$ denotes the costs for a commuter of using road j if T commuters use this road. (So this costs are the same for all commuters who take the road.)

Questions

Questions we want to answer (at least):

- How the commuters will behave?
- Is this behaviour social optimal?
- Is it (Pareto) efficient?

We shall answer these questions by looking to these questions from a game theoretical perspective using game theory.

We assume that the commuters are rational and intelligent. Rationality here concerns that commuters want to minimise costs.

Game structure

We further also refer to the possible routes as strategy and to a commuter as player. We label the players by $1, 2, \ldots, n$.

Let x_i be the strategy of player i. Thus $x_i = 1$ (route 1) or $x_i = 2$ (route 2). Denote by $(x_1, \ldots, x_n)$ a strategy profile, i.e. player 1 plays (i.e. chooses) x_1, player 2 plays x_2, ..., player n plays x_n.

We suppose that the commuters simultaneously and independently choose a route. This will make that we in fact are dealing with non-cooperative game theory.

Analysis

First let us suppose $n = 2$, i.e. that there are 2 commuters. Denote by $C_1(x_1, x_2)$ the total costs of commuter 1 if this commuter chooses strategy x_1 and commuter 2 strategy x_2. Define $C_2(x_1, x_2)$ in the same way.

For example: at the strategy profile $(2, 1)$ (i.e. player 1 takes route 2 and player 2 takes route 1),
player 1 has costs $8 \cdot 1$ for road 3 and $\frac{8}{3}1^2 + \frac{16}{3} = 8$ for road 4.
Thus $C_1(2, 1) = 8 + 8 = 16$.
And for player 2 this leads to $C_2(2, 1) = 2 + 2 = 4$.

Analysis (ctd)

We find
$C_1(1,1) = 13,\ C_2(1,1) = 13$
$C_1(1,2) = 4,\ C_2(1,2) = 16$
$C_1(2,1) = 16,\ C_2(2,1) = 4$
$C_1(2,2) = 32,\ C_2(2,2) = 32$

This can be represented as follows by means of a bimatrix:

$$\begin{pmatrix} 13;13 & 4;16 \\ 16;4 & 32;32 \end{pmatrix}.$$

$$\begin{pmatrix} 13; 13 & 4; 16 \\ 16; 4 & 32; 32 \end{pmatrix}.$$

A simple game theoretic analysis shows

Prediction of behaviour : both choose route 1.

Social optimal : each commuter chooses a different route.

We see:

Equilibrium is not social optimal; This is a typical result.
However, equilibrium is Pareto efficient; this result is not typical.

New course

(New course) in period 4, 2024: Game Theory.

You are very welcome!

Comparing with text book model

Let us compare this congestion model now with the model from the Text Book.

1. General transport network instead of one single freeway and some alternate routes.
2. Finite many players instead of an infinite number.
3. Equilibrium may be or may be not a social optimum; depends on network and costs.
4. We can address Braess' paradox. (See below.)

Model setup

It should be clear from the motivating example, how one can set up a general congestion model. In an abstract way this looks as follows.

- Each player chooses (simultaneously and independently) a particular combination of resources out of a common set of resources.
- With each resource is associated a (may be player specific) cost that depends on the number of players who include it in their choice. (So for this cost it does not matter which players are using a specific resource, only how many players are using it.)
- The total cost for a player is the sum of the costs associated with the resources included in his choice.

Fundamental result

There are various results about congestion games, like the existence of Nash equilibria. The first one was:

Theorem

Rosenthal; A Class of Games Possessing Pure-Strategy Equilibria; International Journal of Game Theory; 1973.

Pioneers concerning congestion games: Rosenthal, Milchtaich, Monderer, and Shapley (Nobel Prize Economics).

What is the paradox about?

The Braess' Paradox is named after the mathematician Dietrich Braess. It states that adding (removing) a link to a transportation network can increase (decrease) the travel cost for all commuters in the network. It is a counterintuitive phenomenon.

The paradox occurs only in networks in which the commuters operate independently and non cooperatively, in a decentralized manner.

Braess' paradox has been observed in various cities, for example in Seoul, New York and Stuttgart. In New York the often congested 42nd was closed for a parade. People suspected that the closing of this road would lead to the worst traffic jams in history. Instead, the traffic flow actually improved that day.

Braess' Paradox (ctd.)

In fact the Braess' Paradox is not limited to traffic flow. It also occurs in other types of "networks". In fact it is widespread occurring for example with biological or electricity systems. This makes this paradox extra interesting!

Example from sport: removing a key player from a basketball team can result in the improvement of the team's offensive efficiency. ('When less is actually more.')

The Braess' paradox may arise as Nash equilibria have not to be 'optimal'.

Braess' Paradox (ctd.)

Let us now look to the following Youtube video:
https://www.youtube.com/watch?v=cALezV_Fwi0

You can deal with this paradox by means of an agent based modelling Netlogo program that You can find in the library of models that is integrated in the Netlogo program. Besides therein are other programs dealing with congestion.

Further reading

Yaron Hollander and Joseph N. Prashker, the applicability of non-cooperative game theory in transport analysis, Transportation (2006) 33:481-496.

What is NetLogo?

NetLogo is a well-written, easy-to-install, easy-to-use (?), easy-to-extend and easy-to-publish-online modelling environment.

It has been developed by Wilansky in 1999 and is designed for coding and running agent-based simulations. There are various alternatives, but NetLogo is the most widely used.

Using and learning NetLogo

NetLogo has a good documentation. In particular there is the User Manual (to be found under the Help tab) that includes three tutorials to help beginners get started.

- Download and install NetLogo following the instructions at `https://ccl.northwestern.edu/netlogo/`.
- Read and work through Tutorial 1 (Models), Tutorials 2 (Commands) and 3 (Procedures) in the NetLogo User Manual.

There also is an extensive library of models from different disciplines.

Using and learning NetLogo (ctd.)

For seriously learning NetLogo, i recommend the (online-)book: Agent Based Evolutionary Game Dynamics,
`https://open.umn.edu/opentextbooks/textbooks/683`

Hotelling Game

Here we consider another example of a model dealing with location: a game theoretical one. It is a discrete variant of the original so-called Hotelling Game.

The (Discrete) Hotelling Game is a game among $n \geq 2$ players that depends on a parameters m, being a positive integer

Consider the $m + 1$ points of $H := \{0, 1, \dots, m\}$ on the real line, to be referred to as *vertices*.

Rule of the game when $n = 2$: 2 players simultaneously and independently choose a vertex. If player 1 (2) chooses vertex x_1 (x_2), then the payoff $f_i(x_1, x_2)$ of player i is the number of vertices that is the closest to his choice x_i; however, a shared vertex, i.e. one that has equal distance to both players, contributes only $1/2$.

Hotelling Game (ctd.)

Various (economic) interpretations of this game are possible: can You provide one?

An answer: Imagine a stretch of beach on which two ice cream retailers want to sell ice cream. The flavours they offer and the prices they charge are the same, so sunbathers go to the closest cart. The question for the two retailers is, where should they set up their carts to get the most customers? In fact there are various variants of this model. Above it is assumed that there is a finite number (i.e. $m + 1$) positions where the sunbathers can enjoy there live.

Hotelling Game (ctd).)

Example $m = 7$.

Strategy profile (5,2) :

Payoffs:

$1 + 1 + 1 + 1 = 4$

$1 + 1 + 1 + 1 = 4$

Strategy profile (0,3) :

Payoffs

$1 + 1 = 2$

$1 + 1 + 1 + 1 + 1 + 1 = 6$

Hotelling Game (ctd.)

Example $m = 7$.

Strategy profile (2,6) :

Payoffs:

$1 + 1 + 1 + 1 + \frac{1}{2} = 4\frac{1}{2}$

$\frac{1}{2} + 1 + 1 + 1 = 3\frac{1}{2}$

Strategy profile (3,3) :

Payoffs:

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4$

Hotelling Game (ctd.)

General rule of the game: n players simultaneously and independently choose a vertex. If player $i = 1, 2, \ldots, n$ chooses vertex x_i, then his payoff $f_i(x_1, x_2, \ldots, x_n)$ is the number of vertices that is the closest to his choice x_i. However, a shared vertex, i.e. one that has the same distance to other players, say k, contributes only $1/(k+1)$.