Urban Economics and Analysis

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Exercises B

Exercise 1 *The following true/false questions deal with a general bimatrix game.*

a. A bimatrix game concerns a game with two players.

b. Each bimatrix game has at least one Nash equilibrium.

c. Each bimatrix game has a social optimum.

d. Each bimatrix game has a Pareto efficient strategy profile.

e. Each social optimum is Pareto efficient.

f. It is impossible that a Pareto inefficient strategy profile is a Nash equilibrium.

Exercise 2 *The following true/false questions deal with the bimatrix game*

$$\begin{pmatrix} 3;6 & 6;5 & 4;3 \\ 6;2 & 5;3 & 5;4 \end{pmatrix}.$$

a. The row-player has 2 *strategies.*

b. There are 6 *strategy profiles.*

c. Playing row 1 *and column* 1 *is a Nash equilibrium.*

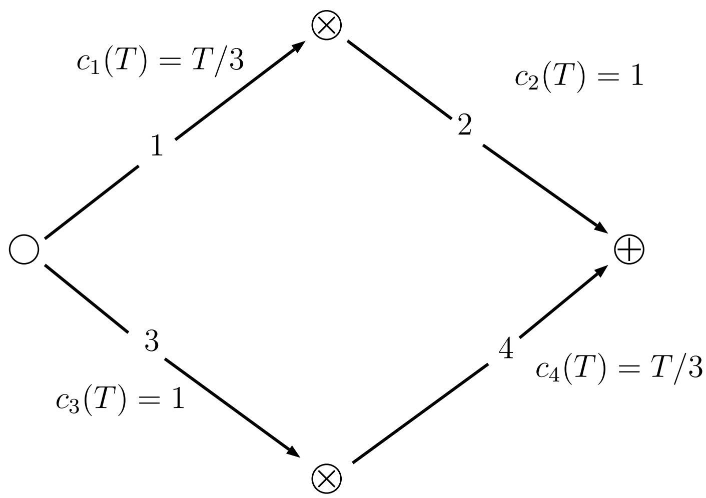
d. There is a Pareto inefficient Nash equilibrium.

e. Playing row 1 *and column* 3 *is a social optimum.*

f. This game is a zero-sum game.

g. Playing row 2 *and column* 1 *is a Pareto efficient strategy profile.*

Exercise 3 *Consider the following variant of the traffic network presented in the context of the Braess' paradox in Slides B. But now with two commuters.*

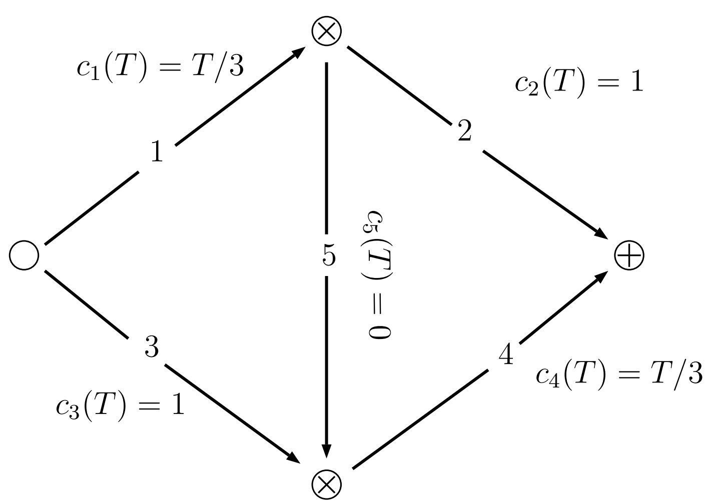


a. *Identify for each commuter the strategies.*

b. *Represent this game as a bimatrix game.*

c. *Determine the Nash equilibria.*

Exercise 4 *Modify the above traffic network by adding as follows a fifth route that can be used without costs.*



d. *Identify for each commuter the strategies.*

e. *Represent this game as a bimatrix game.*

f. *Determine the Nash equilibria.*

g. *Compare with parts c and d in Exercise 3.*

Short solutions.

Solution 1 aT bF cT dT eT fF.

Solution 2 aT bT cF dT eF fF gF.

Solution 3 a. Strategy 1 is route choice $\{1, 2\}$. strategy 2 is route choice $\{3, 4\}$

b. $\begin{pmatrix} 5/3; 5/3 & 4/3; 4/3 \\ 4/3; 4/3 & 5/3; 5/3 \end{pmatrix}$.

c. This game has two Nash equilibria: $(1, 2)$ and $(2, 1)$. In each Nash equilibrium each player has costs $4/3$.

Solution 4 d. Here we have an additional route choice: $1 - 5 - 4$.

e. With the additional route choice as third strategy we obtain the bimatrix game $\begin{pmatrix} 5/3; 5/3 & 4/3; 4/3 & 5/3; 1 \\ 4/3; 4/3 & 5/3; 5/3 & 5/3; 1 \\ 1; 5/3 & 1; 5/3 & 4/3; 4/3 \end{pmatrix}$.

f. This game has a Nash equilibrium: $(3, 3)$. In this equilibrium each player has costs $4/3$.

g. Conclusion: adding route 5 “does not improve the situation”. (This exercise illustrates in a weak way the so-called Braess’ paradox.) Also note: in each Nash equilibrium in part c drivers take a different route while in the unique Nash equilibrium in part f they take the same route.