This is a set of four problems for the classroom or for exercise at home.

Problem 1 *Consider a cost sharing game* (N, c) *with* $N = \{1, 2, 3\}$ *and the cost (charateristic) function c given by*
$c(\{1\}) = 120; c(\{2\}) = 140; c(\{3\}) = 120; c(\{1, 2\}) = 170; c(\{1, 3\}) = 160;$
$c(\{2, 3\}) = 190; c(\{1, 2, 3\}) = 265.$

a) Describe in your own words what function c tells us.

b) Show that the core of (N, c) *is empty.*

Problem 2 *One interpretation of an egalitarian solution in a cost sharing game with subadditive costs is equal sharing of surplus. Consider a cost sharing game* (N, c) *with* $N = \{1, 2\}$ *and the cost (charateristic) function c. The surplus is defined as* $\Delta = c(\{1\}) + c(\{2\}) - c(\{1, 2\})$*. Then egalitarian surplus sharing means* $(x_1^e, x_2^e) = (c(1) - \frac{\Delta}{2}, c(2) - \frac{\Delta}{2})$.

a) Calculate the egalitarian surplus sharing solution when $c(\{1\}) = 120; c(\{2\}) = 140; c(\{1, 2\}) = 170$.

b) Is this solution in the core? Why or why not?

c) How can egalitarian surplus sharing be generalized to three players? And to n players?

d) Consider now $c(\{1\}) = 120; c(\{2\}) = 140; c(\{1, 2\}) = 170$, *as before and a third player such that* $c(\{3\}) = 120; c(\{1, 3\}) = 160; c(\{2, 3\}) = 190; c(\{1, 2, 3\}) = 255$. *Calculate the payoffs for egalitarian surplus sharing.*

e) Show that the solution for d) is not in the core.

f) Find the core of the cost sharing game.

Problem 3 *Consider a two player cost sharing problem with characteristic function* $c(\{1\}) = -120; c(\{2\}) = -140; c(\{1,2\}) = -170$..

a) Describe the situation as a bargaining game, i.e. determine the disagreement point and the bargaining set.

b) Find the Nash bargaining solution.

c) Argue that it must be in the core.

Problem 4 *Consider a cooperative 2-player bargaining problem* (S, d). *Denote a solution to the bargaining problem by* $f((S, d)) = (x_1, x_2)$. *Let* S *be given by the triangle [(0,0),(0,4),(8,0)] in the players' payoff space and let* $d = (1, 2)$.

a) Write down the Nash product.

b) Maximizing the Nash product, subject to $(x_1, x_2) \in S$ *gives* $\frac{9}{8}$. *Use this and the given information to determine the bargaining solution.*

c) Argue why the gains from bargaining are unequally divided.