Game Theory: Assignment 2

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Due date of assignment: **11 March 2025**. Please submit a single pdf file by email.

Consider a river in a arid country where river water is the only source of water. There are two regions, upstream and downstream. Assume the water flow w_0 is fixed and given. Water is productive and will be used in agriculture. Upstream uses water to raise cattle. Downstream uses water to produce crops on irrigated land. Assume that upstream's production function shows diminishing returns to water and can be written as $y_U = \alpha\sqrt{w_U}$. Downstream can expand the irrigated land area. Hence, the production function is $y_D = \beta w_D$ where α and β are productivity parameters. Of course we require that the total water use cannot exceed the available water, $w_U + w_D \leq w_0$. Furthermore, assume that utility is linear in agricultural output for both agents.

a) Describe the water conflict as a non-cooperative game. What is the Nash equilibrium of this game?

b) Now consider water distribution as a cooperative bargaining game. Construct the bargaining set assuming that if there is no agreement, then no one can use water.

c) Derive the Nash Bargaining solution.

d) An investment in water infrastructure increases downstream's water productivity to $\beta' > \beta$ and the available water $w_1 > w_0$. Derive the Nash Bargaining Solution for this case and discuss your findings.

Hint. In order to arrive at numerical solutions you may use your own assumptions for parameters $\alpha, \beta, \beta', w_0$ and w_1.