Game Theory

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Exercise set 5

Exercise 1 *Consider Nim* $(3, 7, 9, 4, 5, 3, 11)$*. First analyse this game and then play it with an opponent.*

Exercise 2 *a. Show that the value of Nim* $(5, 7, 6, 4, 1, 3, 9, 15, 15)$ *is* 1*, i.e. that player* 1 *has a winning strategy. Determine a first move for this player which is optimal.*

b. What is the value of $(5, 3, 8, 6, 7, 4, 3, 1, 1, 3, 4, 7, 6, 8, 3, 5)$*?*

Exercise 3 *Show that the in Lesson 6 given optimal strategies for Nim are correct by showing*

a. After a move, a position with an even Nim-sum becomes a position with an odd Nim-sum.

b. A position with an odd Nim-sum admits a move who leads to a position with an even Nim-sum.

Short solutions.

Solution 1 Nim-sum is 2,3,4,6. So is odd. Player 1 can win.

Solution 2 a. The number of matches in the binary system is 0101, 0111, 0110, 0100, 0001, 0011, 1001, 1111, 1111. This gives as Nim sum $3, 6, 5, 7$. So the Nim sum is odd, meaning that we have to do with a winning position. A first optimal move is to take away 11 matches from the last pillow (resulting in a Nimsom $2, 6, 4, 6$).

b. -1. (This can quickly seen by "symmetry".)

Solution 3 a. Consider a position with an even Nim-sum. The position being even means that all column sums are even. If you now make a move, every column will change at most one digit; where depends on the choice of the pillows where something is taken away. At least there is at least one column where the column sum changes. The addition in that column now produces an odd number and therefore an odd Nimsom.

b. Consider a position with an odd Nim-sum. Find the leftmost column with an odd number of 1's. Now fix a pillow in this column that has a 1 in that column. Then in the base-two number of matches in that column, change that 1 to a 0 and everything else to the right of that 1 such that an even number of 1's occurs in the corresponding columns. This lowers the above base-two number; associated with this is a move that leads to an even Nim-sum.