Game Theory

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Exercise set 3

Exercise 1 *Consider a duopoly in case of two producers with proice function*

$$p(X) = 200 - \frac{1}{4}X$$

and with cost functions

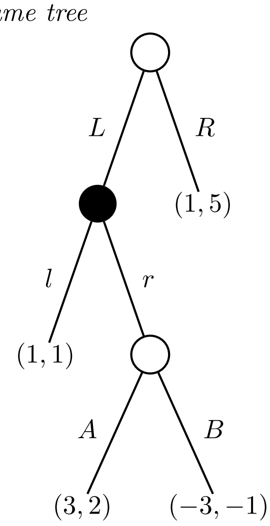
$$c_1(x_1) = 20x_1, \;\; c_2(x_2) = 10x_2.$$

a. Determine the profit functions $\pi_1(x_1, x_2)$ and $\pi_2(x_1, x_2)$.

b. Determine both reaction functions $R_1(x_2)$ and $R_2(x_1)$.

c. Determine the Cournot-equilibrium (i.e. quantities and price).

Exercise 2 *Consider the following 2-player extensive form game given by the game tree*



a. Determine a normalisation of this game.

b. Show that there are four Nash equilibria.

c. Which Nash equilibrium is "the best"?

Exercise 3 *Consider the following game between two (rational and intelligent) players. There is a pillow with* 100 *matches. They alternately remove* $1, 2$ *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. What is the value of this game?*

Short solutions.

Solution 1 a. $\pi_1(x_1, x_2) = 180x_1 - \frac{1}{4}x_1^2 - \frac{1}{4}x_1x_2$, $\pi_2(x_1, x_2) = 190x_2 - \frac{1}{4}x_2^2 - \frac{1}{4}x_1x_2$.
b. $R_1(x_2) = 360 - \frac{1}{2}x_2$, $R_2(x_1) = 380 - \frac{1}{2}x_1$.
c. $x_1 = 680/3, x_2 = 800/3, p = 230/3$.

Solution 2 a. A normal form is $\begin{pmatrix} & l & r \\ LA & 1;1 & 3;2 \\ LB & 1;1 & -3;-1 \\ RA & 1;5 & 1;5 \\ RB & 1;5 & 1;5 \end{pmatrix}$. b. Nash equilibria: $(LA, r), (RA, l), (RB, l)$ and (LB, l).

c. (LA, r). Reason: if player 1 has to move for the second time, then he plays A. Player 2 is aware of this, and therefore, if he has to move, plays r. Player 1 is aware of this and therefore plays L as first move.

Solution 3 The loosing positions are those with number of matches that when divided by 3 has remainder 0. As 100 divided by 3 has remainder 1, player 1 can win. So the value is $+1$.