# Game Theory

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Exercise set 2

**Exercise 1** *Determine which of the following bimatrix games are a prisoner's dilemma.*

*a.* $\begin{pmatrix} 3;-1 & 3;1 & 6;0 \\ 1;0 & 3;1 & 6;0 \\ 2;2 & 4;1 & 8;2 \end{pmatrix}.$

*b.* $\begin{pmatrix} 1;0 & 3;1 & 6;0 \\ 2;1 & 4;1 & 8;1 \end{pmatrix}.$

*c.* $\begin{pmatrix} 6;1 & 3;1 & 1;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}.$

*d.* $\begin{pmatrix} -1;-1 & 2;0 \\ 0;2 & 3;3 \end{pmatrix}.$

*e.* $\begin{pmatrix} 2;2 & -1;3 \\ 3;-1 & 0;0 \end{pmatrix}.$

**Exercise 2** *The following true/false statements concern an arbitrary bimatrix game.*

*a. This concerns a game with two players.*

*b. The game has at least one Nash equilibrium.*

*c. The game has a strictly dominant strategy.*

*d. The game has a fully cooperative strategy profile.*

*e. Each fully cooperative strategy profile is weakly Pareto efficient.*

*f. The game has a weakly Pareto efficient strategy profile.*

*g. A strictly dominant strategy is fully cooperative.*

*h. If the game is a prisoners' dilemma, then it has a Nash equilibrium.*

*i. It is impossible that a weakly Pareto inefficient strategy profile is a Nash equilibrium.*

**Exercise 3** *The following true/false statements deal with the bimatrix game*

$$\begin{pmatrix} 3;6 & 6;5 & 7;-3 \\ -6;2 & 5;3 & 5;4 \end{pmatrix}.$$

*a. The row-player has 2 strategies.*

b. *There are* 6 *strategy profiles.*

c. *The strategy profile* $(1, 1)$ *is a Nash equilibrium.*

d. *The row-player has a strictly dominant strategy.*

e. *There is a weakly Pareto inefficient Nash equilibrium.*

f. *The column-player has a strictly dominant strategy.*

g. *This game is a prisoners' dilemma.*

h. *Playing row* 1 *and column* 3 *is a fully cooperative strategy profile*

i. *This game is a zero-sum game.*

j. $(1, 2)$ *is a weakly Pareto efficient strategy profile.*

**Exercise 4** *A new notion: a strict Nash equilibrium is a Nash equilibrium with the property that if a player deviates from his strategy in this Nash equilibrium, his payoff will become less.*

*Given the following bimatrix game:*

$$\begin{pmatrix} 3;8 & -4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;3 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

a. *Determine the best reply correspondences.*

b. *Determine the strictly dominant strategies.*

c. *Determine the Nash equilibria.*

d. *Determine the strict Nash equilibria.*

e. *Determine the weakly Pareto-efficient strategy profiles.*

**Exercise 5** *Consider the Hotelling Game with sites* $0, 1, 2, 3, 4$.

a. *Why does* $f_1(4 - x_1, 4 - x_2) = f_1(x_1, x_2)$ *hold?*

b. *Show that for the payoff function* $f_1$ *of player* 1 *the following formula holds:*

$$f_1(x_1, x_2) \begin{cases} \frac{x_1+x_2+1}{2} \text{ if } x_1 < x_2, \\ \frac{5}{2} \text{ if } x_1 = x_2, \\ 5 - \frac{x_1+x_2+1}{2} \text{ if } x_1 > x_2 \end{cases}$$

c. *Show that* $(2, 2)$ *is a Nash equilibrium.*

Short solutions.

*Solution 1* Only the game in e.

*Solution 2* aT bF cF dT eT fT gF hT iF.

*Solution 3* aT bT cT dT eF fF gF hF iF jT.

*Solution 4* a. $R_1(1) = \{1, 3\}$, $R_1(2) = \{3\}$, $R_1(3) = \{2\}$, $R_2(1) = \{1, 2\}$, $R_2(2) = \{1\}$, $R_2(3) = \{2\}$, $R_2(4) = \{1\}$.
a. Strictly dominant strategies: do not exist.
b. They are $(1, 1)$ (i.e. row 1 and column 1) and $(3, 2)$.
c. $(3, 2)$.
e. $(1, 1), (1, 2)(2, 3), (3, 2)$.

*Solution 5* a. Because there is a "location symmetry".
b. Just calculate by hand the bimatrix and verify this formula.
c. Analyse the bimatrix in part b.