Urban Economics and Analysis

Part of P. v. Mouche

Assignment B; 2022-2023

Exercise 1 *The following true/false questions deal with the bimatrix game*

$$\begin{pmatrix} 3;6 & 6;5 & 4;3 \\ 6;2 & 5;3 & 5;4 \end{pmatrix}.$$

a. The row-player has 2 *strategies.*

b. There are 6 *strategy profiles.*

c. Playing row 1 *and column* 1 *is a Nash equilibrium.*

d. There is a Pareto inefficient Nash equilibrium.

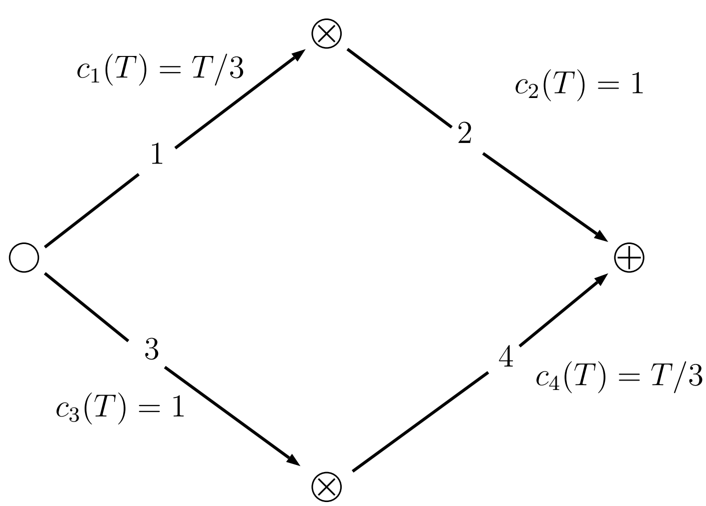
e. Playing row 1 *and column* 3 *is a social optimum.*

f. This game is a zero-sum game.

g. Playing row 2 *and column* 1 *is a Pareto efficient strategy profile.*

h. Playing row 1 *and column* 1 *is a social optimum.*

Exercise 2 *Consider the following variant of the traffic network with two commuters presented in the context of the Braess' paradox in Slides B.*

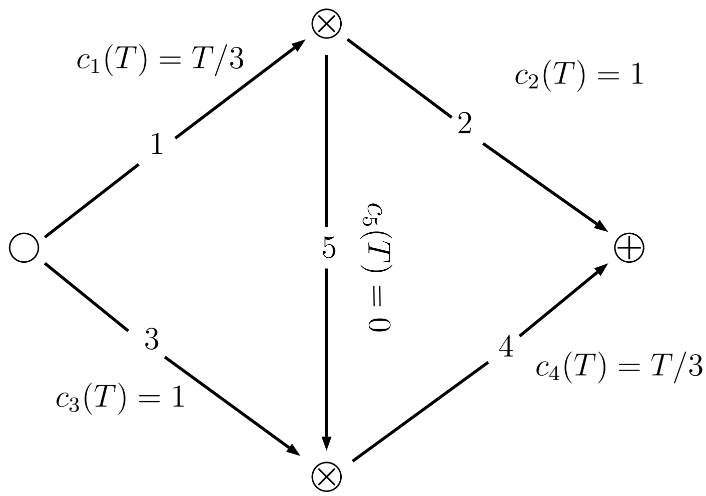


a. *Identify for each commuter the strategies.*

b. *Represent this game as a bimatrix game.*

c. *Determine the Nash equilibria.*

Exercise 3 *Modify the above traffic network by adding as follows a fifth route that can be used without costs.*



d. *Identify for each commuter the strategies.*

e. *Represent this game as a bimatrix game.*

f. *Determine the Nash equilibria and the price of anarchy.*

g. *Compare with parts c and d in Exercise 3.*

Exercise 4 *'Play' with the following two NetLogo programs.*

- *Traffic Basic (Choose File/Models Library/Social Science/Traffic Basic).*
- *Braess' Paradox (Send to You by email).*

Short solutions.

Solution 1 aT bT cF dT eF fF gF hF.

Solution 2 a. Strategy 1 is route choice $1-2$. Strategy 2 is route choice $3-4$.

b. $\begin{pmatrix} 5/3; 5/3 & 4/3; 4/3 \\ 4/3; 4/3 & 5/3; 5/3 \end{pmatrix}$.

c. This game has two Nash equilibria: $(1,2)$ and $(2,1)$. In each Nash equilibrium each commuter has costs $4/3$.

Solution 3 d. Here we have an additional route choice: $1-5-4$.

e. With the additional route choice as third strategy we obtain the bimatrix game $\begin{pmatrix} 5/3; 5/3 & 4/3; 4/3 & 5/3; 1 \\ 4/3; 4/3 & 5/3; 5/3 & 5/3; 1 \\ 1; 5/3 & 1; 5/3 & 4/3; 4/3 \end{pmatrix}$.

f. This game has a Nash equilibrium: $(3,3)$. In this equilibrium each commuter has costs $4/3$.

g. Conclusion: adding route 5 “does not improve the situation”. (This exercise illustrates in a weak way the so-called Braess’ paradox.) Also note: in each Nash equilibrium in part c drivers take a different route while in the unique Nash equilibrium in part f they take the same route.

Solution 4