Advanced microeconomics

2022

1 Math recall questions

Exercise 1 *Determine the derivative of the following functions:*

a. $f(x) = 4x^3 + 11$.

b. $f(x) = \sqrt{x}$.

c. $f(x) = ax^b$.

d. $f(x) = \ln(x)$.

e. $f(x) = 2x^2 \ln(x)$.

f. $f(x) = (a^2 + x^2)e^x$.

g. $f(x) = \sqrt{1 - x^2}$.

h. $f(x) = \frac{x^2}{x+7}$.

i. $f(x) = xp(x + 2a)$. *(Here p is a function.)*

Exercise 2 *Determine the partial derivative of the following functions with respect to x_1:*

a. $f(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$.

b. $f(x_1) = u(x_1, m - \frac{p_1 x_1}{p_2})$.

Exercise 3 *Determine the maximisers and the maximum of the following functions functions.*

a. $f(x) = -x^2 + 4x + 7$.

b. $f(x) = e^{-(x^2-1)^2}$.

Exercise 4 *Determine an antiderivative of the following functions:*

a. $f(x) = x^2$

b. $f(x) = ax^b$ *where* $b \neq -1$.

And determine:

c. $\int_{-1}^{0} (2x^3 - 6x)\mathrm{d}x$.

d. $\int_{1}^{-1} 7 \;\mathrm{dx}$.

e. $\int_{1}^{\infty} 2/x^2 \mathrm{d}x$.

Exercise 5 *Solve the two equations*

$$\frac{p_1}{p_2} = \frac{\alpha_1}{\alpha_2}\frac{x_2}{x_1}, \; p_1x_1 + p_2x_2 = m$$

for x_1 *and* x_2. *(Here* p_1, p_2 *and* m *are parameters).*

2 Microeconomics recall questions

Exercise 6 *Give a demand function* $Q(p)$ *for which the price elasticity of demand is (everywhere)* -3.

Exercise 7 *Suppose the equation* $k_1 + \sqrt{k_2} = 9$ *is an isoquant of a production function. Determine the marginal rate of substitution of production factor* 1 *for production factor* 2 *in an arbitrary production factor bundle* (k_1, k_2) *on the isoquant.*

Exercise 8 *Consider the total cost function* $c(q) = a - bq^3 + 2dq$.

a. Determine the marginal cost function and average variable cost function.

b. Why, in the formula of $c(q)$, b *should be negative?*

Exercise 9 *Consider the Cobb-Douglas utility function* $u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$.

a. Give the formula for the Marshallian demand functions, i.e. the functions that describe the utility maximising good bundle as a function of the prices p_1, p_2 *and income* m.

b. Give another utility function with the same Marshallian demand functions.

Exercise 10 *Consider the utility function* $u(x_1, x_2) = \min(x_1, x_2)$.

a. *Which types of goods are represented by this utility function?*

b. *Determine the Marshallian demand functions.*

Exercise 11 *Determine the net consumer surplus for the inverse demand function* $p(Q) = 6 - 2Q$ *at price* $p \in [0, 6]$.

Exercise 12 *Consider the Cobb-Douglas production function* $f(k_1, k_2) = Ak_1^{\beta_1} k_2^{\beta_2}$.

a. *Determine the marginal rate of substitution of production factor* 1 *for production factor* 2 *in the production factor bundle* $(k_1, k_2) = (3, 4)$.

b. *What type of returns to scale has this production function?*

Exercise 13 *Determine the cost function for the Cobb-Douglas production function* $f(k_1, k_2) = k_1^{1/3} k_2^{2/3}$ *in case of production factor prices* $w_1 = 1$ *and* $w_2 = 2$.

Exercise 14 *Determine the Nash equilibria (in pure strategies) for the bi-matrix-game*

$$\begin{pmatrix} 5;5 & 4;0 & 1;9 \\ 3;0 & 0;6 & 2;10 \\ 7;8 & 5;11 & 3;-3 \end{pmatrix}.$$

Exercise 15 *Consider a duopoly in case of two producers with inverse market demand function*

$$p(Q) = 200 - \frac{1}{4}Q$$

and with cost functions

$$c_1(q_1) = 20q_1, \quad c_2(q_2) = 10q_2.$$

a. *Determine both reaction functions* $R_1(q_2)$ *and* $R_2(q_1)$.

b. *Determine the Cournot-equilibrium (i.e. quantities and price).*

Solution 1 a. $12x^2$. b. $1/(2\sqrt{x})$. c. abx^{b-1}. d. $1/x$. e. $4x \cdot \ln x + 2x^2/x = 2x(2 \ln x + 1)$. f. $(2x)e^x + (a^2 + x^2)e^x = (a^2 + 2x + x^2)e^x$. g. $\frac{-x}{\sqrt{1-x^2}}$. h. $\frac{2x(x+7)-x^2}{(x+7)^2} = \frac{x^2+14x}{(x+7)^2}$. i. $p'(x + 2a)x + p(x + 2a)$.

Solution 2 a. $\alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2}$.

b. $\frac{\partial u}{\partial x_1} u(x_1, m - \frac{p_1 x_1}{p_2}) - \frac{p_1}{p_2} \frac{\partial u}{\partial x_2}(x_1, m - \frac{p_1 x_1}{p_2})$.

Solution 3 a. Only 2 is a maximiser. The maximum is 15.

b. Only 0 is a maximisers are -1 and 1. The maximum is 0.

Solution 4 a. $\frac{1}{3}x^3 + 137$.

b. $\frac{a}{b+1}x^{b+1}$.

c. $5/2$.

d. -14

e. 2.

Solution 5 $x_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} \frac{m}{p_1}$, $x_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} \frac{m}{p_2}$.

Solution 6 $Q(p) = p^{-3}$.

Solution 7 The isoquant is $k_2 = (9 - k_1)^2$. So the desired rate is $-\frac{dk_2}{dk_1} = 2k_1 - 18$.

Solution 8 a. The marginal cost function equals $c'(q) = -3bq^2 + 2d$. The variable cost function is $-bq^3 + 2dq$. So the average variable cost function is $-bq^2 + 2d$.

b. Because a total cost function is increasing.

Solution 9 a. $\tilde{x}_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} \frac{m}{p_1}$, $\tilde{x}_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} \frac{m}{p_2}$.

b. $u(x_1, x_2) = 931 x_1^{\alpha_1} x_2^{\beta_2} + 137$.

Solution 10 a. Perfect complements.

b. $\tilde{x}_1(p_1, p_2; m) = \frac{m}{p_1+p_2}$, $\tilde{x}_2(p_1, p_2; m) = \frac{m}{p_1+p_2}$.

Solution 11 This is the area of the region under the inverse demand function where prices are at least p. It is a triangle and equals $\frac{1}{2}(6 - p)(\frac{6-p}{2})$.

Solution 12 a. This rate equals $\frac{\partial f}{\partial k_1} / \frac{\partial f}{\partial k_2} = \frac{\beta_1}{\beta_2} \frac{k_2}{k_1}$. So in $(3, 4)$ it equals $\frac{4\beta_1}{3\beta_2}$.

b. If $\beta_1 + \beta_2 < 1$, then diminishing returns to scale. If $\beta_1 + \beta_2 = 1$, then constant returns to scale. If $\beta_1 + \beta_2 > 1$, then increasing returns to scale.

Solution 13 $c(q) = 3q$.

Solution 14 There is a unique Nash-equilibrium: it is the multi-strategy $(3, 2)$, i.e third row for player 1 and second column for player 2.

Solution 15 a. $R_1(q_2) = 360 - \frac{1}{2}q_2$, $R_2(q_1) = 380 - \frac{1}{2}q_1$.

b. $q_1 = 680/3, q_2 = 800/3, p = 230/3$.