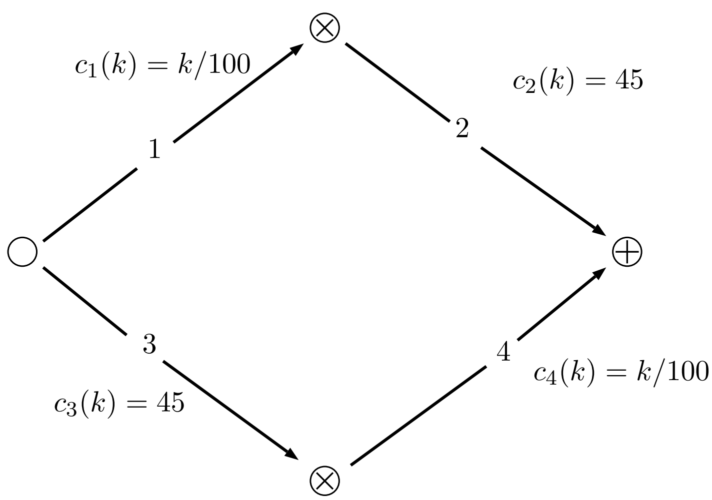
Advanced Microeconomics

P. v. Mouche

Assignment for part 2

This assignment concerns a congestion game. See Lesson 6 for a description of this game.

Consider the following traffic network in case of n commuters, where n is even.



As in Lesson 6, route 1 concerns roads 1-2 and route 2 concerns roads 3-4. Again denote total costs for commuter i in a route profile $\mathbf{x} = (x_1, \ldots, x_n)$ by $C_i(\mathbf{x})$. So for each commuter i, $x_i = 1$ or $x_i = 2$.

Given $\mathbf{x} = (x_1, \ldots, x_n)$, let $n_1(\mathbf{x})$ be the number of commuters who choose route 1 and $n_2(\mathbf{x})$ the number of commuters who choose route 2.

1. Show that for each commuter i

$$C_i(\mathbf{x}) = \begin{cases} c_1(n_1(\mathbf{x})) + c_2(n_1(\mathbf{x})) \text{ if } x_i = 1; \\ c_3(n_2(\mathbf{x})) + c_4(n_2(\mathbf{x}) \text{ if } x_i = 2. \end{cases}$$

2. Show that a route profile $\mathbf{e}$ satisfying $n_1(\mathbf{e}) = n_2(\mathbf{e})$ is a Nash equilibrium. Is such a Nash equilibrium strict? (Hint: determine the costs $C_1(\mathbf{e})$ and $C_2(\mathbf{e})$ and the costs if one player chooses his other possible route in $\mathbf{e}$).

3. Determine all Nash equilibria.

Please handle (by email) in before October 14.