Microeconomics

Please remember

Autumn 2018

"The time has come," the Walrus said, "To talk of many things:
Of shoes - and ships - and sealing-wax - Of cabbages - and kings
And why the sea is boiling hot - And whether pigs have wings."
(Lewis Carroll)

1. Prices and income.
   - Prices: p_1 and p_2. Income: m
   - Prices always are supposed to be positive. (Handling prices 0 is difficult.)
   - Budget restriction (and budget set): $p_1x_1 + p_2x_2 \leq m$.
   - Budget line: $p_1x_1 + p_2x_2 = m$. Thus $x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$.
   - A value tax t replaces a price p by $(1+t)p$. A quantity tax t replaces a price p by $p+t$.
2. Indifference sets.
   - The sets of good bundles for which the consumer is indifferent are called indifference sets. Remark: these sets often are, *par abus de langage*, called curves.
   - Indifference sets cannot intersect.
3. Utility.
   - Under weak conditions (You do not need to know) a preference relation $\succeq$ can be represented by a utility function u. This means that

     $$(x_1, x_2) \succsim (y_1, y_2) \text{ is equivalent with } u(x_1, x_2) \geq u(y_1, y_2),$$

     $$(x_1, x_2) \succ (y_1, y_2) \text{ is equivalent with } u(x_1, x_2) > u(y_1, y_2),$$

     $$(x_1, x_2) \sim (y_1, y_2) \text{ is equivalent with } u(x_1, x_2) = u(y_1, y_2),$$

   - Utility is ordinal in the sense that each strictly increasing transformation of the utility function describes the same economic situation.
   - Only cardinal utility later when we deal with uncertainty.
   - Specific (important) utility functions.
     - Cobb-douglas: $u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$.
     - Solow: $u(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2$. (Perfect substitutes.)
     - Leontief: $u(x_1, x_2) = \min(x_1/\alpha_1, x_2/\alpha_2)$. (Perfect complements.)
     - Maximum: $u(x_1, x_2) = \max(x_1/\alpha_1, x_2/\alpha_2)$.
     - Quasi-linear: $u(x_1, x_2) = v(x_1) + x_2$. (Good 1 is quasi-linear.)
     - Special quasi-linear: $u(x_1, x_2) = \alpha\sqrt{x_1} + x_2$.
   - Indifference.
     - An indifference curve $x_2(x_1)$ is such that $u(x_1, x_2(x_1)) = \text{constant}$.
     - Marginal rate of substitution at a good bundle:[1]

       $$\text{MRS} := \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}.$$

       In case $x_2(x_1)$ is an indifference curve,

       $$\text{MRS} = -\frac{dx_2}{dx_1}.$$

       For cobb-douglas:

       $$\text{MRS} = \frac{\alpha_1}{\alpha_2}\frac{x_2}{x_1}.$$

     - For homothetic preferences the MRS is constant along each ray through the origin.

[1] In the book and elsewhere in economics also MRS is defined as $-\frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$, i.e. with a minus sign.

- For quasi-linear utility functions the indifference curves are vertical translates of each other.

- Method of Lagrange.
  - Optimisation problem: maximise (or minimise) a function $f(x_1, \ldots, x_n)$ under m restrictions $g_i(x_1, \ldots, \ldots, x_n) = 0$ $(1 \leq i \leq m)$ (for our applications we always have $m = 1$.)
  - Lagrange function $L = f - \lambda_1 g_1 - \cdots - \lambda_m g_m$.
  - In optimum: $\frac{\partial L}{\partial x_1} = 0, \cdots, \frac{\partial L}{\partial x_n} = 0$.
- Two laws of Gossen.
  - Gossen's first law about diminishing marginal utility is nonsense. But decreasing MRS makes sense.
  - Gossen's second law: $p_1/p_2 = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$ (see below).
  - Gossen's second law can be proved graphically or by the method of Lagrange.
- Marshallian demand functions.
  - They are the solutions of the following utility maximisation problem: maximise $u(x_1, x_2)$ under the budget restriction $p_1 x_1 + p_2 x_2 \leq m$.
  - Denoted by $\tilde{x}_1(p_1, p_2; m), \ \ \tilde{x}_2(p_1, p_2; m)$.
  - They can be determined by solving

$$p_1 x_1 + p_2 x_2 = m,$$

    and an additional condition. These conditions are:
    * For cobb-douglas and quasi-linear: $p_1/p_2 = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$ (Gossen's second law).
    * For leontief: being at a kiknk-point, i.e. $x_1/\alpha_1 = x_2/\alpha_2$ holds (optimum at kink).
    * For Solow: optimum is one boundary point or whole budget line is optimal.
    * For Maximum: optimum is one or two boundary points.
  - For cobb-douglas:

$$\tilde{x}_1(p_1, p_2; m) = \frac{\alpha_1}{\alpha_1 + \alpha_2} \frac{m}{p_1}, \ \ \tilde{x}_2(p_1, p_2; m) = \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{m}{p_2}.$$

  - Attention: the marshallian demand of a quasi-linear good is independent of the income if this income is large enough (and for such incomes often can be determined with Gossens's second law).

4. Types of goods.

- Consider a price change or income change of good i. Good i is called
  - giffen, if $p_i \uparrow \ \Rightarrow \ \ \tilde{x}_i \uparrow$. In a formula: $\frac{\partial \tilde{x}_i}{\partial p_i} > 0$.
  - ordinary, if $p_i \uparrow \ \Rightarrow \ \ \tilde{x}_i \downarrow$. In a formula: $\frac{\partial \tilde{x}_i}{\partial p_i} < 0$.
  - normal, if $m \uparrow \ \Rightarrow \ \ \tilde{x}_i \uparrow$. In a formula: $\frac{\partial \tilde{x}_i}{\partial m} > 0$.
  - inferior, if $m \uparrow \ \Rightarrow \ \ \tilde{x}_i \downarrow$. In a formula: $\frac{\partial \tilde{x}_i}{\partial m} < 0$.
- Consider two goods i and j. Good i is called
  - a substitute for good j, if $p_j \uparrow \ \Rightarrow \ \ \tilde{x}_i \uparrow$.
  - a complement for good j, if $p_j \uparrow \ \Rightarrow \ \ \tilde{x}_i \downarrow$.

5. Elasticity.

- Setting: variable b depends on variable a, i.e. a function $b(a)$.
- Elasticity is measure for how sensible b depends on a. Elasticity should be independent of units. Two types of elasticity: segment and point elasticity. We only deal with point elasticities.

- Formula for point elasticity
  $$\epsilon = \frac{db}{da}\frac{a}{b}.$$
  Interpretation: relative change of b divided by relative change of a.
- - $|\epsilon| < 1$: inelastic; $|\epsilon| > 1$: elastic.
  - In case of an affine function, i.e. $q = \alpha p + \beta$ (with $\alpha < 0$ and $\beta > 0$) and p on the ordinate, each point above middle of graph is elastic and each point below is inelastic. In middle the elasticity is -1.
  - Good to know: $b = Aa^{\alpha}$ has $\epsilon = \alpha$.
  - $\epsilon = \frac{d \ln b}{d \ln a}$.
- Consider the price elasticity of demand: $\epsilon = \frac{dq}{dp}\frac{p}{q}$.
  - Revenue $R(p) = p \cdot q(p)$.
  - $\frac{dR}{dp}\frac{p}{R} = 1 + \epsilon$.
  - If inelastic, then $p \uparrow \Rightarrow \ R \uparrow$.
  - If elastic, then $p \uparrow \Rightarrow \ R \downarrow$.
- Perfect inelastic demand curve: curve is vertical. Perfect elastic demand curve: curve is horizontal.
- Consider a marshallian demand function $\tilde{x}_i$. Income elasticity of demand $\eta_i = \frac{\partial \tilde{x}_i}{\partial m}\frac{m}{\tilde{x}_i}$.
  - Luxury good i: $\eta_i > 1$.
  - Necessary good i: $\eta_i < 1$.

6. Market demand and partial equilibrium.
   - Setting: decreasing demand function $D(p)$ and increasing supply function $S(p)$.
   - Inverse demand function $P(Q)$.
   - Without taxes equilibrium price $p^{\star}$ is determined by
     $$S(p^{\star}) = D(p^{\star}).$$
   - Equilibrium with taxes:
     - Two prices: p_D (price buyer pays) and p_S (price seller gets). In case of a positive quantity tax: $p_D = p_S + t$:
     - equilibrium prices determined by
       $$p_D^{\star} = p_S^{\star} + t, \ \ D(p_D^{\star}) = S(p_S^{\star});$$
     - result $p_S^{\star} \leq p^{\star} \leq p_D^{\star}$;
     - $p_D^{\star} - p^{\star}$ is part of tax passed along the consumer; it is the smaller the more elastic D and the more inelastic S.
       $p^{\star} - p_S^{\star}$ is part of tax passed along the producer; it is the smaller the more elastic S and the more inelastic D.
   - Deadweight loss is the loss of consumers' and produces' surplus due to the tax.
7. Returns to scale for production function $f(k_1, k_2)$.
   - Constant returns to scale $f(tk_1, tk_2) = tf(k_1, k_2)$ for all $t > 1$.
     Increasing returns to scale $f(tk_1, tk_2) > tf(k_1, k_2)$ for all $t > 1$.
     Deceasing returns to scale $f(tk_1, tk_2) < tf(k_1, k_2)$ for all $t > 1$.
   - - For cobb-douglass production function $Ak_1^{\beta_1}k_2^{\beta_2}$:
       $\beta_1 + \beta_2 = 1$: constant returns to scale.
       $\beta_1 + \beta_2 > 1$: increasing returns to scale.
       $\beta_1 + \beta_2 < 1$: decreasing returns to scale.

  - Leontief and solow production functions have constant returns to scale.

8. Cost minimisation.

   - Isoquant: set of production factor bundles with same output
   - Technical rate of substitution:
   $$\text{TRS} := \frac{\partial f}{\partial k_1} / \frac{\partial f}{\partial k_2}.$$
   In case $k_2(k_1)$ is an isoquant,
   $$\text{TRS} = -\frac{dk_2}{dk_1}.$$
   - Conditional production factor demand functions
   $$\tilde{k}_1(w_1, w_2; q), \quad \tilde{k}_2(w_1, w_2; q)$$
   are the solutions of the cost minimisation problem. Each of the above specific production functions has its own solving conditions for the cost minimisation problem. One condition always is
   $$f(k_1, k_2) = q.$$
   Additional conditions:
     - Cobb-douglas and quasi-linear: $f(k_1, k_2) = q, \quad w_1/w_2 = \frac{\partial f}{\partial k_1} / \frac{\partial f}{\partial k_2}$.
     - For leontief: being at a kink-point.
     - For Solow: optimum is one boundary point or whole isoquant is optimal.
     - For Maximum: optimum is one or two boundary points.
   - Cost function: $c(q; w_1, w_2) = w_1\tilde{k}_1(w_1, w_2; q) + w_2\tilde{k}_2(w_1, w_2; q)$.
   - Various cost functions: fixed costs $c(0)$, marginal costs $\frac{dc}{dq}$, variable costs $c(q) - c(0)$, average costs $\frac{c(q)}{q}$, average variable costs $\frac{c(q)-c(0)}{q}$.
   - Increasing returns to scale leads to decreasing average costs. Decreasing returns to scale leads to increasing average costs. Constant returns to scale leads to constant average costs.
   - Principle of 'the marginal leads the average': as long as the marginal is larger than the average, the average increases and as long as the marginal is less than the average, the average decreases.
   - In the short term some production factors are fixed. In the long term there are no fixed costs.

9. Profit maximisation.

   - profit maximisation implies cost minimisation.
   Input perspective: profit function $\pi(k_1, k_2) = pf(k_1, k_2) - (w_1k_1 + w_2k_2)$.
   Output perspective: profit function $\Pi(q) = pq - c(q; w_1, w_2)$.
   Both perspectives give the same results; this is intuitively clear (but not formally).
   - Production factor functions
   $$\overline{k}_1(p; w_1, w_2), \quad \overline{k}_2(p; w_1, w_2)$$
   are the solutions of the profit maximisation problem. For input perspective they can for cobb-douglas production functions (and other well behaved functions) be determined by solving:
   $$p\frac{\partial f}{\partial k_1} = w_1, \quad p\frac{\partial f}{\partial k_2} = w_2.$$
   And for output perspective by solving:
   $$p = c'(q).$$
   - Profit maximisation is (for arbitrary prices) not compatible with constant and with increasing returns to scale. (This one can easily check for the production function $f(k) = k^\alpha$.)
   - Short term: Let $p^\star$ be the minimum of the average variable cost curve: this is the shutdown price. Supply curve is marginal cost curve for $p \geq p^\star$. For $p < p^\star$ supply is zero.