

Advanced Microeconomics

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Assignment part 2

For the assignment You may choose between two tasks, Task 1 and Task 2. Task 1 concerns the exercise on the next page. Task 2 concerns to pick first a game out of the “Play garden of games” on the course web page under “Additional”, next to make a little proposal concerning Your motivation for this game and what You would like to do for the assignment with it. Next You send me this proposal and i will decide whether it is ok (may be with making some modifications of the proposal). If You choose for Task 1, then please handle it in before 7 October 2022. If You choose for Task 2, then please handle it in before 13 October 2022.

We are going to define a game in strategic form with two player where player i has payoff function $f_i : H \times H \rightarrow \mathbb{R}$.

Consider the $m+1$ points of $H = \{0, 1, \dots, m\}$ on the real line, to be referred to as *vertices*. Two players simultaneously and independently choose a vertex. If player 1 chooses vertex x_1 and player 2 vertex x_2 , then the payoffs are as follows.

Case $w = 1$: the payoff of player i , $f_i(x_1, x_2)$, is the number of vertices that is the closest to his choice x_i ; however, a vertex that has equal distance to both players contributes only $1/2$.

General $w \in [0, 1]$: to $f_i(x_1, x_2)$ exactly the same vertices as in the above for $w = 1$ contribute. Take such a vertex. If it is at distance d to x_i , then it contributes w^d if it is not a shared vertex, and otherwise it contributes $w^d/2$.

Thus this assignment deals with the (discrete) Hotelling game. If You like, reconsider Lesson 1 for some examples.

1. Suppose m is even and $w = 1$. (Attention: You should deal with a general odd m .)
 - a. Determine the best reply correspondences R_1 and R_2 .
 - b. Determine the Nash equilibria.
 - c. Determine the weakly and strongly Pareto efficient strategy profiles.
2. Suppose $m = 4$ and $w = 1/2$.
 - a. Determine the game by representing it as a 5×5 -bi-matrix game with at the first row strategy 0, the second row strategy 1, etc.
 - b. Determine the best reply correspondences R_1 and R_2 .
 - c. Determine the Nash equilibria
 - d. Determine the weakly and strongly Pareto efficient strategy profiles.
3. Suppose $m = 2$ and $w = 1/2$. But now suppose that first player 1 makes his choice and then player 2 after having observed the choice of his opponent.
 - a. Present the game tree.
 - b. Determine the subgame perfect Nash equilibria.