Advanced Microeconomics

P. v. Mouche

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Consider the $m+1$ points of $H = \{0, 1, \dots, m\}$ on the real line, to be referred to as *vertices*. Two players simultaneously and independently choose a vertex. If player 1 chooses vertex x_1 and player 2 vertex x_2, then the payoffs are as follows.

<u>Case $w = 1$</u>: the payoff of player i, $f_i(x_1, x_2)$, is the number of vertices that is the closest to his choice x_i; however, a vertex that has equal distance to both players contributes only $1/2$.

<u>General $w \in [0, 1]$</u>: to $f_i(x_1, x_2)$ exactly the same vertices as in the above for $w = 1$ contribute. Take such a vertex. If it is at distance d to x_i, then it contributes w^d if it is not a shared vertex, and otherwise it contributes $w^d/2$.

Thus this assignment deals with the Hotelling game. If You like, reconsider Lesson 1 for some examples.

1. Suppose m is odd and $w = 1$.
   a. Determine the best reply correspondences R_1 and R_2.
   b. Determine the Nash equilibria.
   c. Determine the weakly and strongly Pareto efficient strategy profiles.
2. Suppose $m = 4$ and $w = 1/2$.
   a. Determine the game by representing it as a 5×5-bi-matrix game with at the first row strategy 0, the second row strategy 1, etc.
   b. Determine the best reply correspondences R_1 and R_2.
   c. Determine the Nash equilibria
   d. Determine the weakly and strongly Pareto efficient strategy profiles.
3. Suppose $m = 2$ and $w = 1/2$. But now suppose that first player 1 makes his choice and then player 2 after having observed the choice of his opponent.
   a. Present the game tree.
   b. Determine the subgame perfect Nash equilibria.

Please handle in before 9 October 2021.