Game Theory

Lesson 1: Motivation and Outlook

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Organisation

Welcome to the Game Theory course!

- Lecturers: P. v. Mouche (weeks 1 and 2) and H.-P. Weikard (week 3).
- Content: a very short introduction to game theory.
- There are two parts. Part 1: non-cooperative game theory (weeks 1,2). Part 2: cooperative game theory (week 3).
- Course is designed for all students interested in decision-making in strategic situations.
- Starting level: ’no’ required knowledge. We focus on concepts and shall only use simple mathematics.

Concerning first part

My slides are self-contained.

Besides working seriously through these slides, read related content in the little book

'Game Theory, a Very Short Introduction' of the game theorist K. Binmore.

For the first part this concerns Chapters 1 and 3. Doing this will enhance Your understanding and appreciation for the topics involved.

Organisation of first part

So let us start with the first part. I’ll teach this part in five Lessons.

Lesson 1: Motivation and Outlook.
Lesson 2: Bimatrix Games.
Lesson 3: Games in Strategic Form.
Lesson 4: Games in Extensive Form.
Lesson 5: Congestion.

What You will learn

After studying this lesson, i.e. Lesson 1, You

- should be able to explain what game theory is about;
- should be familiar with the concrete games introduced in this lesson;
- should know which real-world types of games one distinguishes.

In addition:

- You should be familiar with the specific games introduced in this lesson, i.e. Tic-Tac-Toe, Hex and Nim.

What is game theory?

Traditional game theory deals with mathematical models of conflict and cooperation in the real world between at least two rational intelligent players.

- Player: humans, organisations, nations, animals, computers, ...
- Situations with one player are studied by the classical optimisation theory.
- 'Traditional' because of rationality assumption.

Nature of game theory

- Applications.
  - Economics: Nobel prices in 1994 for Nash, Harsanyi and Selten, in 2005 for Aumann and in 2007 for Meyerson and Maskin.
  - Sociology, psychology, antropology, politocology.
  - Military strategy.
  - Biology (evolutionary game theory).
  - Design of computer games and robots.
- Game theory provides a language that is very appropriate for conceptual thinking.
- Many game theoretical concepts can be understood without advanced mathematics.
- Aim of game theory is to understand/predict how games will be played.

Players

In general we shall denote the players by numbers. And in the case of n players by $1, \ldots, n$.

Further on, when dealing with theory, we often deal for simplicity with two players: player 1 and player 2, or white and black, ... (In practice, for parlor games, a device like a die may decide who is which player.)

Outcomes and payoffs

- A game can have different outcomes, i.e. ways the game can be played. Each outcome has its own payoffs for every player.
- Interpretation of payoff: 'satisfaction' at end of game.
- Nature of payoff: money, honour, activity, nothing at all, utility, real number, ...
- In many parlor games with two players, the payoff of a player can be represented as winning, draw or loosing.
- In general it does not make sense to speak about 'winners' and 'losers' (and/or 'draw'). It does, however, in various parlour games, like Chess, Tic-Tac-Toe and Football.

Rationality and intelligence

- Because there is more than one player, especially rationality becomes a problematic notion. Here is a simple try: a player is rational if he has well-defined preferences concerning the outcomes of the game.
- Intelligence also is a not so easy notion. It presupposes an intelligent player and refers to the (rational) goal of that player. Intelligence has to do with the way the goal is approached.
- So rationality' and 'intelligence' are different concepts and the intelligence notion presupposes which type of rationality we are speaking about.
- In many games rationality is not a big assumption.

Concrete games

We shall consider various concrete games in order to illustrate the abstract theory with that we develop. These games concern Parlor games (this lesson):

- Tic-Tac-Toe.
- Hex.
- Nim.

Economic games (next lessons):

- Cournot Oligopoly.
- Hotelling Game.
- Congestion Game.

Concrete games (ctd.)

Parlor games have strict rules. But economic games are game theoretic models (so also have rules) of real-world economic situations where rules are not strict.

In the above parlor games, (ordinary) rationality means: winning is better than draw and draw is better than loosing.

Tic-Tac-Toe

- Tic-Tac-Toe is a very well-known game.
- Here You can play this game online: `https://papergames.io/en/tic-tac-toe/`. Please do it several times.
- The game has many (more than three) outcomes. However, only three types of outcomes: player 1 wins, draw, player 1 loses.
- Example of a play of this game:

Tic-Tac-Toe (ctd.)

X		

X		
		O

X	X	
		O

X	X	O
		O

X	X	O
X		O

X	X	O
		O
X		O

So: player 2 is the winner.

Question: Is player 1 intelligent? Is player 1 rational?
Answer: We do not know. However, if player 1 is rational, then he is not intelligent. And, if player 1 is intelligent, then he is not rational. Reason: each player can obtain at least a draw.

Hex

Please see

1. Here Yo can play the Hex `http://www.lutanho.net/play/hex.html`. Please do it several times.
2. The game on the above web page has an 11×11 board. In fact one can play Hex also for other board sizes.
3. Hex was invented independently by Piet Hein and John Nash.

Hex (ctd.)

The Hex game has very interesting properties:

1. No Hex game can end in a draw.
   This statement is equivalent with Brouwer's fixed point theorem in two dimensions, being a deep mathematical result.
2. Player 1 always can win the game. (Later we shall be able to see that this is true.)
3. If You can give a winning strategy for (player 1 for) each Hex game, then essentially You solved one of the seven so-called Millennium problems (each worth 1 million dollar). (Have a look to `https://en.wikipedia.org/wiki/Millennium_Prize_Problems` if You like.)

Nim

Nim is the following two player game. A certain number of piles consisting of a certain number of matches is put together. Both players take turns. Player 1 starts. Each turn a player must remove at least one match from a pile. The player that takes the last matche(s) wins. So there are infinitely many Nim games possible.

Consider Nim $(2, 3, 5, 5, 3, 2)$, i.e. there is one pillow with 2 matches, one with 3, ..., and one with 2. Play this game with an opponent. Be sure that You see how You can win this game if You are player 2.

Also play Nim $(5, 7, 6, 4, 1, 3, 9)$.

Real-world types

In order to set up a theory for games one has to specify how the games that one considers relate to the real-word. (In red what we will study when we develop theory.)

- all players are rational – players may be not rational
- all players are intelligent – players who may be not intelligent
- binding agreements – no binding agreements
- chance moves –no chance moves
- communication – no communication
- static game – dynamic game
- transferable payoffs – no transferable payoffs

Real-world types (ctd.)

- interconnected games – isolated games
- perfect information – imperfect information
- complete information – incomplete information

The choices we made in red are very appropriate for dealing with non-cooperative game theory. Cooperative game theory willt be dealt with in the second part of the course.

Below we briefly will consider some of these notions.

Perfect information

- A player has perfect information if he knows at each moment when it is his turn to move how the game was played until that moment.
- A player has imperfect information if he does not have perfect information.
- A game is with (im)perfect information if (not) all players have perfect information.
- Chance moves are compatible with perfect information.
- Examples of games with perfect information: Tic-Tac-Toe, Hex, Chess, ...
  Examples of games with imperfect information: many card games like Poker, and Monopoly (because of the cards, not because of the die).

Complete information

- A player has complete information if he knows all payoff functions.
- A player has incomplete information if he does not have complete information.
- A game is with (in)complete information if (not) all players have complete information.
- Examples of games with complete information:
  Tic-Tac-Toe, Chess, Poker, Monopoly, ...
  Examples of games with incomplete information: auctions, oligopoly models where firms only know the own cost functions, ...

Common knowledge

Something is common knowledge if everybody knows it and in addition that everybody knows that everybody knows it and in addition that everybody knows that everybody knows that everybody knows it and ...

Common knowledge is a very interesting subject, however not so easy to grasp it. It is quite related to mathematical logic.

Common knowledge

A group of dwarfs with red and green caps are sitting in a circle around their king who has a bell. In this group it is common knowledge that every body is intelligent. They do not communicate with each other and each dwarf can only see the color of the caps of the others, but does not know the color of the own cap. The king says: ”Here is at least one dwarf with a red cap.”. Next he says: “I will ring the bell several times. Those who know their cap color should stand up when i ring the bell.”. Then the king does what he announced.

Common knowledge (ctd.)

The spectacular thing is that there is a moment where a dwarf stands up. Even, when there are M dwarfs with red caps that all these dwarfs simultaneously stand up when the king rings the bell for the M-th time.

Do not worry if You do not see why this claim this true. Dealing with such things is very advanced and quite important for the fundamental basis of game theory. However, it is to advanced for our simple Game Theory course. But if You like to know why the claim is true, then please have a look at

`https://en.wikipedia.org/wiki/Common_knowledge_(logic)`

where a similar situation is dealt with.

Common knowledge (ctd.)

What do we learn from this? Well, loosely speaking:

- Suppose in a group of people everybody knows something, say fact X holds. If somebody in this group says that fact X holds, then this may add a lot of information: X becomes common knowledge.
- Thus something becomes common knowledge just by speaking about it.

Appetizer

The time has come to start with developing theory for predicting how games, like the above ones, will (or better said ‘may’) be played by rational intelligent players. Understanding this is our ultimate goal.

Among other things we shall: (try to) make clear that games like Tic-Tac-Toe, Hex and Nim have a so-called value. (Rigorously) proving this is too involved for us in this simple Game Theory course.
Also we explain what can be said about how games without value will be played, like the Hotelling Game, Cournot Oligopoly, Congestion Game and many other economic games

The most important object in ‘magic box’ for achieving this is the Nash equilibrium