Game Theory

P. v. Mouche

Exercise set 5

Exercise 1 *Consider a Cournot duopoly with price function* $p(X) = 10 - X$ *and cost functions* $c_1(x_1) = x_1^2$ *and* $c_2(x_2) = x_2$.

a. Determine the profit functions.

b. Determine the Cournot equilibrium.

c. Determine the equilibrium price.

Exercise 2 *Consider a Cournot duopoly with price function* $p(X) = 200 - \frac{1}{4}X$ *and cost functions* $c_1(x_1) = 20x_1, \;\; c_2(x_2) = 10x_2$. *Are the foloowing statements true or false?*

a. A strategy x_i *of a firm* i *represents a price.*

b. This is a prisoners' dilemma game.

c. This is a finite game.

d. This is a zero-sum game.

e. If firm plays $x_1 = 200$ *and firm* 2 *plays* $x_2 = 400$*, then the payoff of firm* 1 *is* 6000.

f. $(680/3, 800/3)$ *is a Nash equilibrium.*

g. The equilibrium price is $230/3$.

Exercise 3 *a. Convert the following decimal numbers into binary ones: 100, 200, 16384.*

b. Convert the following binary numbers into a decimal one: 101010101010

c. Convert the binary number 11001000 into a hexadecimal one:

Exercise 4 *Consider Nim* $(3, 7, 9, 4, 5, 3, 11)$. *First analyse this game and then play it with an opponent.*

Exercise 5 *a. Show that the value of Nim* $(5, 7, 6, 4, 1, 3, 9, 15, 15)$ *is* 1*, i.e. that player* 1 *has a winning strategy. Determine a first move for this player which is optimal.*

b. *What is the value of* $(5, 3, 8, 6, 7, 4, 3, 1, 1, 3, 4, 7, 6, 8, 3, 5)$*?*

Exercise 6 *Show that the given optimal strategies for Nim are correct by showing*

a. *After a move, a position with an even Nim-sum becomes a position with an odd Nim-sum.*

b. *A position with an odd Nim-sum admits a move who leads to a position with an even Nim-sum.*

Short solutions.

Solution 1 a. $\pi_1(x_1, x_2) = 10x_1 - 2x_1^2 - x_1x_2$.
$\pi_2(x_1, x_2) = 9x_2 - x_2^2 - x_1x_2$.
b. Solving $\frac{\partial \pi_1}{\partial x_1} = 0$ together with $\frac{\partial \pi_2}{\partial x_2} = 0$ gives $x_1 = 11/7$ and $x_2 = 26/7$
c. 33/7.

Solution 2 a. F. b. F. c. F. d. F. e. T. f. T. g. T.

Solution 3 a. 1100100, 11001000, 100000000000000.
b. 2730.
c. C8.

Solution 4 Nim-sum is 1,3,4,6. So is odd. Player 1 can win.

Solution 5 a. The number of matches in the binary system is 0101, 0111, 0110, 0100, 0001, 0011, $1001, 1111, 1111$. This gives as Nim sum $3, 6, 5, 7$. So the Nim sum is odd, meaning that we have to do with a winning position. A first optimal move is to take away 11 matches from the last pillow (resulting in a Nimsom $2, 6, 4, 6$).
b. -1. (This can quickly seen by "symmetry".)

Solution 6 a. Consider a position with an even Nim-sum. The position being even means that all column sums are even. If you now make a move, every column will change at most one digit; where depends on the choice of the pillows where something is taken away. At least there is at least one column where the column sum changes. The addition in that column now produces an odd number and therefore an odd Nimsom.

b. Consider a position with an odd Nim-sum. Find the leftmost column with an odd number of 1's. Now fix a pillow in this column that has a 1 in that column. Then in the base-two number of matches in that column, change that 1 to a 0 and everything else to the right of that 1 such that an even number of 1's occurs in the corresponding columns. This lowers the above base-two number; associated with this is a move that leads to an even Nim-sum.