Game Theory

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Exercises 1

Exercise 1 *This exercise deals with the Tic-Tac-Toe game. Lesson 1 contains a link where You can play the game. For the first two parts of this exercise You need an opponent.*

a. *Play the game several times with an opponent. Be sure that (after some plays, may be also at home) You can play this game in such a way that You do not loose.*

b. *Both players write on a paper a completely elaborated plan of play, i.e. write down how they will play the game. Then they put both papers on the table and play the game using these strategies. Note: the plan of play should be completely clear: say a third person (called 'referee') can play the game for You, without asking You what You mean or taking a decision how to play.*

c. *Now suppose You are player* 1 *(i.e. You begin). Try to find a completely elaborated plan of play that guarantees You at least a draw.*

Exercise 2 *This exercise deals with the Hex game. For the first part of this exercise You need an opponent. (Lesson 1 contains a link where You can play the game against a computer program.)*

a. *Play the game various times with an opponent.*

b. *As explained in Lesson 1, it impossible (and quite surprising) that this game cannot end in a draw (i.e. no winner, no loser). Please convince Yourself that this statement is true by trying to obtain a draw.*

Exercise 3 *Consider Tic-Tac-Toe (with the standard numbering of cells). Consider the following completely elaborated plan of play player* 1.

First move in cell 5. *Each following move according to the first description in the following list that can be applied:*

(1) Lowest number in same row in which opponent did last move.
(2) Lowest number in same column in which opponent did last move.
(3) Lowest number.

Give a completely elaborated plan of play of player 2 *that wins from that of player* 1.

Exercise 4 *Given the following bi-matrix-game:*

$$\begin{pmatrix} 3;8 & 4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;4 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

a. *Determine the strictly dominant strategies.*

b. *Determine the Nash equilibria.*

c. *Determine the weakly Pareto efficient strategy profiles.*

d. *Determine the fully cooperative strategy profiles.*

Short solutions.

Solution 1 c. Number the cells from left to right and from top to bottom with $1, 2, \ldots, 9$ First move: 5. For each next move, if it applies, move opposite to last own move (and then win), otherwise move clockwise beside last move of opponent and if this is not possible, then move anti-clockwise beside last move of opponent.

Solution 2 I hope that You enjoyed trying.

Solution 3 Consider the following completely elaborated plan of play: $(3, 5, 1, 9, 7, 6, 2, 4, 8)$, i.e. player moves in first free cell number that occurs in this sequence. Player 2 wins.

Solution 4 a. No player has a strictly dominant strategy.
b. Nash equilibria: $(1, 1)$ (i.e. row 1 and column 1), $(1, 2), (3, 1), (3, 2)$.
c. $(1, 1), (1, 2), (2, 3), (3, 2)$.
d. $(1, 2)$.