# Autumn 2019

## Fundamental exercises

## Microeconomics

Below You find some fundamental exercises. Do not forget to make also some of the the selected exercises from the workbook (see webpage).

**Exercise 1** *False or true?*

a. *If* $x^2 = 1$, *then* $x = -1$.

b. *If* $f(x) = x^2 + \frac{17}{x}$, *then* $f'(x) = 2x - \frac{17}{x^2}$.

c. *If* $f(x) = \ln(x) + 9$, *then* $f'(x) = 1/x$.

d. *If* $f(x) = \sqrt{x}$, *then* $f'(x) = 1/(2\sqrt{x})$.

e. *If* $f(x) = e^{3x}$, *then* $f'(x) = 3e^{3x}$.

f. *If* $f(x) = x\ln(x)$, *then* $f'(x) = 1 + \ln(x)$.

g. *If* $f(x) = x \cdot p(x)$, *then* $f'(x) = p(x) + x \cdot p'(x)$.

h. *If* $f(x) = \frac{x^2+1}{2x}$, *then* $f'(x) = \frac{x^2-1}{2x^2}$.

i. *if* $f'(x) = 2x$, *then* $f(x) = x^2$.

j. *If* $f(x, y) = x^2y^{-7} + y$, *then* $\frac{\partial f}{\partial x} = 2xy^{-7}$.

k. *If* $f(x, y, w) = (w - x - y)(3w + 2x - 4y)$, *then* $\frac{\partial f}{\partial y} = -7w + 2x + 8y$.

l. *If* $x < y$, *then* $-x > -y$.

m. *If* $x > 0$, *then* $x^2 \geq x$.

n. $8^{4/3} = 16$.

o. $\frac{40^{1/3}}{4} = (\frac{5}{8})^{1/3}$.

p. *If* $f(x, y) = x^\alpha y^\beta$, *then show that* $\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y} = \frac{\alpha}{\beta}\frac{y}{x}$.

q. *If* $f(x) = Ax^\alpha$, *then show that* $\frac{f'(x)x}{f(x)} = \alpha$.

r. *If* $f(x) = \sqrt{2x}$, *then* $f'(x) = \frac{1}{2x}$.

**Exercise 2** a. *Solve the two equations* $x + 2y = 7,\ 3x + 6y = 9$ *in* $x, y$.

b. *Solve the two equations* $3 = 5\frac{x_2}{x_1},\ \ 6x_1 + 2x_2 = 9$ *in* $x_1, x_2$.

c. *Solve the two equations*

$$\frac{p_1}{p_2} = \frac{\alpha_1}{\alpha_2}\frac{x_2}{x_1},\ \ p_1x_1 + p_2x_2 = m$$

*in* $x_1, x_2$. *(Here* $p_1, p_2$ *and* $m$ *are parameters.)*

**Exercise 3** *Analyse for the following relations $R$ on a set $X$ their reflexivity, completeness and transitivity.*

*a.* $X = \mathbb{R}$, *the set of real numbers, and $R$ is the relation $\geq$;*

*b.* $X = \mathbb{R}$ *and $R$ is the relation $\leq$;*

*c.* $X = \mathbb{R}$ *and $R$ is the relation $>$;*

*d.* *$X$ is the set of people with the Dutch nationality and $xRy$ is defined by $x$ is a friend of $y$.*

*e.* $X = \{1, 2, 3, \ldots\}$ *and $xRy$ means $x$ divides $y$.*

**Exercise 4** *Consider the utility function $u(x_1, x_2) = x_1 x_2^2$.*

*a. Determine the indifference curve $x_2(x_1)$ through the good bundle $(4, 2)$. What is $dx_2/dx_1$ at $(4, 2)$?*

*b. Determine the marginal rate of substitution, i.e. $MRS = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$. What is its value at $(4, 2)$?*

*c. Compare the results in a and b.*

**Exercise 5** *Determine the marginal rate of subsitution of the the utility function $u_1(x_1, x_2) = x_1 + 2x_2$ and the marginal rate of subsitution of the utility function $u_2(x_1, x_2) = (x_1 + 2x_2)^2 + 137$. Explain what You observe.*

**Exercise 6** *Determine the marshallian demand functions for the leontief utility function $u(x_1, x_2) = \min(3x_1, 4x_2)$.*

**Exercise 7** *Consider a consumer with the utility function*

$$u(x_1, x_2) = \sqrt{x_1} + x_2.$$

*a. Show that the consumer strictly prefers $(16, 2)$ to $(4, 3)$.*

*b. Show that the consumer does not strictly prefer $(144, 18)$ to $(36, 27)$.*

*c. Is $u$ homothetic? (Remember: $u$ is homothetic means $u(x_1, x_2) \geq u(y_1, y_2) \Rightarrow u(\lambda x_1, \lambda x_2) \geq (\lambda y_1, \lambda y_2)$ for all $\lambda > 0$.)*

*d. Determine the utility maximising good bundle in case $p_1 = 1, p_2 = 2, m = 44$.*

*e. Determine the marshallian demand functions if the income $m$ is large /(enough).*

**Exercise 8** *Consider the utility function*

$$u(x_1, x_2) = 7 \ln(x_1 + 1) + x_2.$$

*a. Determine the marginal rate of substitution.*

*b. Determine the marshallian demand functions (in case the income is not too small).*

*c. Determine the optimal good bundle for the following cases.*

- $p_1 = 1, p_2 = 4, m = 103.$
- $p_1 = 1, p_2 = 4, m = 10.$

**Exercise 9** *Consider the utility function*

$$u(x_1, x_2) = -x_1^2 + 100x_1 + x_2 + \frac{x_2^2}{2}.$$

*Prove that good 1 is giffen by analysing the utility maximisation problem for the price change $p_1 = 1$ and $p_1{}' = 11/10$, assuming that $p_2 = 1$ and $m = 55$. (You may do this by using Gossen's second law: 'price ratio equals marginal rate of substitution'.)*

**Exercise 10** *a. Determine the elasticity of the function* $f(x) = x^7$.

*b. Determine the elasticity of the function* $f(x) = e^x$ *at* $x = 0$.

**Exercise 11** *Suppose a consumer consuming three types of goods has the following marshallian demand function for good* 1

$$\tilde{x_1}(p_1, p_2, p_3; m) = 50 - \frac{24p_1}{p_3} + \frac{6p_2^2}{p_1 p_3} + \frac{m}{1000p_2}.$$

*Determine for good* 1 *its* $p_1$*-elasticity, its* $p_2$*-elasticity, its* $p_3$*-elasticity and its* $m$*-elasticity in the case where* $p_1 = 4$, $p_2 = 2$, $p_3 = 3$ *and* $m = 10000$.

**Exercise 12** *Determine the equilibrium quantity in the case of a demand function* $D(p) = 1000 - 10p$, *a supply function* $S(p) = -100 + 2p$.

*a. Determine the equilibrium price and the equilibrium quantity in the case there is no tax.*

*b. Determine the equilibrium prices and the equilibrium quantity in the case of a quantity tax* $t = 3$.

*c. Determine in b also the part of tax passed along the consumer and the part of tax passed along the producer.*

*d. Calculate the deadweight loss.*

**Exercise 13** *Determine the type of returns to scale of the following production functions.*

*a.* $f(k_1, k_2) = \sqrt{k_1} + \sqrt{k_2}$.

*b.* $f(k_1, k_2) = (k_1^\rho + k_2^\rho)^{1/\rho}$.

**Exercise 14** *a. Determine the cost minimizing production factor bundle for the production function* $f(k_1, k_2) = k_1^2 k_2$ *in case* $w_1 = 1, w_2 = 2$ *and* $q = 10$.

*b. Determine the cost minimizing production factor bundle for the production function* $f(k_1, k_2) = \sqrt{k_1} + \sqrt{k_2}$ *in case* $w_1 = 1, w_2 = 2$ *and* $q = 10$. *(This problem can be handled with 'price ratio = marginal rate of substitution'.)*

*c. Determine for the production function* $f(k_1, k_2) = \min(2k_1, 3k_2)$ *the conditional production factor functions and the cost function.*

**Exercise 15** *Consider the production function*

$$f(k_1, k_2) = k_1 k_2^2.$$

*Suppose production factor prices are* $w_1 = 1$ *and* $w_2 = 2$, *and the output price is* $p = 4$.

*a. Determine the the profit maximising production factor bundle and output via the input perspective. Do this by solving the two equations which state that the value of the marginal product of a production factor equals its price.*

*b. Determine the the profit maximising production factor bundle and output via the output perspective. (So do this by first determining the cost function* $c(q)$*.)*

**Exercise 16** *Consider the production function*

$$f(k_1, k_2) = k_1^{1/4} k_2^{1/2}.$$

*Suppose production factor prices are* $w_1 = 1$ *and* $w_2 = 2$, *and the output price is* $p = 8$.

a. *Determine the the profit maximising production factor bundle and output via the input perspective. Do this by solving the two equations which state that the value of the marginal product of a production factor equals its price.*

b. *Determine the the profit maximising production factor bundle and output via the output perspective. (So do this by first determining the cost function $c(q)$.)*

**Exercise 17** *Determine the shut down price in case of the short term cost function* $c(q) = 2q^3 - 16q^2 + 64q + 50$.

**Exercise 18** *Joep's von-neumann-morgenstern utility function for sure outcome $c$ is given by* $U(c) = c^2$.

a. *Is Joep risk loving?*

b. *Calculate* $U((c_1, \pi_1), \ldots, (c_n, \pi_n))$.

*Further suppose that Joep has 100 euro. He can take a card from a (standard) card game. If he gets diamonds, then he obtains 20 euro and thus has 120 euro. Otherwise, he has to pay 20 euro and thus has 80 euro.*

c. *Will Joep participate in the gamble?*

**Exercise 19** *Socrates is an expected utility maximiser with von neumann-morgenstern utility function* $U(c) = \sqrt{c}$. *Socrates owns just one ship. The ship is worth 200 million Euro. If the ship sinks, Socrates loses 200 million Euro. The probability that it will sink is 2/100. Socrates' total wealth, including the value of the ship is 225 million Euro. What is the maximum amount that Socrates would be willing to pay in order to be fully insured against the risk of losing his ship?*

**Exercise 20** *A risk-averse individual is offered a choice between a gamble that pays 1000 euro with probability 1/4 and 100 euro with probability 3/4, or a payment of 325 euro.*

a. *Calculate the expected value and the expected utility of the gamble.*

b. *Which choice will he make?*

**Exercise 21** *A new notion: the indirect utility function $v(p_1, p_2; m)$ is the function that gives the value of the maximal utility level the consumer can reach at prices $p_1, p_2$ and income $m$.*

*Determine the indirect utility function $v(p_1, p_2; m)$ for the leontief utility function* $u(x_1, x_2) = \min(3x_1, 4x_2)$.

**Exercise 22** *Determine the compensating variation* CV *for the utility function* $u(x_1, x_2) = \min(x_1, x_2))$ *if the price of good 1 changes from 1 to 4, in case $p_2 = 3$ and $m = 15$.*

**Exercise 23** *Determine* CV, EV *and* $\Delta S$ *for the utility function*

$$u(x_1, x_2) = \sqrt{x_1} + x_2$$

*if the price of good 1 changes from 1 to 2, in case $p_2 = 2$ and $m = 20$.*

**Exercise 24** *A new notion: a strategy profile is fully cooperative if it maximises the total payoff.*

*The following true/false questions deal with games in strategic form.*

a. *A bi-matrix-game concerns a game with two players.*

b. *Each bi-matrix-game has at least one nash equilibrium.*

c. *Each bi-matrix-game has a (strictly) dominant strategy.*

d. *Each bi-matrix-game has a fully cooperative strategy profile.*

e. *Each bi-matrix-game has a (weakly) pareto efficient strategy profile.*

f. *Each fully cooperative strategy profile is (weakly) pareto efficient.*

g. *A strictly dominant strategy is fully cooperative.*

h. *A prisoners' dilemma game has a nash equilibrium.*

i. *It is impossible that a (weakly) pareto inefficient strategy profile is a Nash equilibrium.*

j. *A nash equilibrium is a strategy profile that consists of strategies of the players' that they like the most.*

**Exercise 25** *The following true/false questions deal with the bi-matrix-game*

$$\begin{pmatrix} 3;6 & 6;5 & 7;-3 \\ -6;2 & 5;3 & 5;4 \end{pmatrix}.$$

a. *The row-player has* 2 *strategies.*

b. *There are* 6 *strategy profiles.*

c. *Playing row* 1 *and column* 1 *is a Nash equilibrium.*

d. *The row-player has a (strictly) dominant strategy.*

e. *There is a (weakly) pareto inefficient nash equilibrium.*

f. *The column-player has a (strictly) dominant strategy.*

g. *This game is a prisoners' dilemma.*

h. *Playing row* 1 *and column* 3 *is a fully cooperative strategy profile*

i. *This game is a zero-sum game.*

j. *Playing row* 1 *and column* 2 *is a (weakly) pareto efficient strategy profile.*

**Exercise 26** *Determine the (strictly) dominant strategies, the nash equilibria, the fully cooperative strategy profiles and the (weakly) pareto efficient strategy profiles for the following bi-matrix-games. Also determine which games are prisoners' dilemma games.*

a. $\begin{pmatrix} 5;5 & 4;0 & 1;9 \\ 3;0 & 0;6 & 2;10 \\ 7;8 & 5;11 & 3;-3 \end{pmatrix}.$

b. $\begin{pmatrix} 5;5 & -4;6 \\ 6;-4 & -3;3 \end{pmatrix}.$

c. $\begin{pmatrix} 1;1 & 0;0 \\ 0;0 & 1;1 \end{pmatrix}.$

d. $\begin{pmatrix} 5;10 & 6,9 \\ 6,11 & 6,12 \end{pmatrix}.$

e. $\begin{pmatrix} 8;4 & 1;6 & -4;3 \\ 3;-3 & 4;1 & 3;-2 \\ 7;-1 & 9;5 & 2;1 \end{pmatrix}.$

f. $\begin{pmatrix} 3;3 & 0;6 \\ 6;0 & 1;1 \end{pmatrix}.$

g. $\begin{pmatrix} 1;0 & 6;1 & 0;7 \\ 2;4 & 0;2 & 3;3 \\ 3;9 & 2;0 & 4;0 \end{pmatrix}$.

h. $\begin{pmatrix} 5;5 & 4;0 & 7;9 \\ 3;0 & 0;6 & 2;10 \\ 7;8 & 5;11 & 3;-3 \end{pmatrix}$.

**Exercise 27** *Consider a duopoly in case of two producers with inverse market demand function*

$$p(Q) = 200 - \frac{1}{4}Q$$

*and with cost functions*

$$c_1(q_1) = 20q_1, \quad c_2(q_2) = 10q_2.$$

a. *Determine the profit functions $\pi_1(q_1, q_2)$ and $\pi_2(q_1, q_2)$.*

b. *Determine both reaction functions $R_1(q_2)$ and $R_2(q_1)$.*

c. *Determine the Cournot-equilibrium (i.e. quantities and price).*

d. *Determine the Von-Stackelberg-equilibrium when 1 is the leader.*

e. *Determine the collusion equilibrium. (Attention: first think!)*

**Exercise 28** *Consider a pure exchange economy with two consumers $A$ and $B$ and two goods 1 and 2. The utility functions are*

$$u^A(x_1, x_2) = x_1x_2, \; u^B(x_1, x_2) = x_1^2x_2.$$

*$A$ has 4 units of good 1 and 2 units of good 2; $B$ has 3 units of good 1 and 3 units of good 2.*

a. *Write down the equations for a pareto efficient allocation.*

b. *Determine a pareto efficient allocation.*

**Exercise 29** *Consider a pure exchange economy with two consumers $A$ and $B$ and two goods 1 and 2. The utility functions are*

$$u^A(x_1, x_2) = x_1x_2,$$
$$u^B(x_1, x_2) = \min(x_1, x_2).$$

*$A$ has 5 units of good 1 and nothing of good 2; $B$ has 6 units of good 2 and nothing of good 1.*

a. *Determine the walrasian demand functions for good 1 for each consumer.*

b. *Determine the aggregated excess demand for good 1.*

c. *Determine the equilibrium price $(p_1, p_2)$ in case $p_1 = 1$.*

**Exercise 30** *Consider a pure exchange economy with two consumers $A$ and $B$ and two goods 1 and 2. The utility functions are*

$$u^A(x_1, x_2) = x_1x_2 + 2x_1 + 5x_2,$$

$$u^B(x_1, x_2) = x_1x_2 + 4x_1 + 2x_2.$$

*$A$ has 78 units of good 1 and nothing of good 2; $B$ has 164 units of good 2 and nothing of good 1.*

a. *Determine the marshallian demand functions for each consumer (by using price ratio equals marginal rate of substitution).*

*b. Determine the excess demands for each consumer.*

*c. Determine the aggregated excess demands for each of the goods.*

*d. Determine the equilibrium prices and equilibrium quantities.*

*e. Check that the equilibrium allocation in d is pareto efficient (by calculating the marginal rates of substitution).*

SHORT SOLUTIONS. You have to fill in the details yourself.

*Solution 1* All statements are true with exception of a, i, m and r.

*Solution 2* a. No solution.
b.$x_1 = 5/4,\ x_2 = 3/4$.
c. $x_1 = \frac{\alpha_1}{\alpha_1+\alpha_2}\frac{m}{p_1}$ and $x_2 = \frac{\alpha_2}{\alpha_1+\alpha_2}\frac{m}{p_2}$.

*Solution 3* a. Reflexive, complete, transitive.
b. Reflexive, complete, transitive.
c. Not reflexive, not complete, transitive.
d. Not reflexive, not complete, not transitive.
e. Reflexive, not complete, transitive.

*Solution 4* a. As $u(4,2) = 16$, we obtain $x_2(x_1) = 4/\sqrt{x_1}$. This gives $dx_2/dx_1 = -2x_1^{-3/2}$; at $(4,2)$ this equals $-1/4$.
b. $MRS = \frac{1}{2}\frac{x_2}{x_1}$. At $(4,2)$ this equals $1/4$.
c. Theory tells us that $\frac{dx_2}{dx_1} = -MRS$, in accordance with what we found in a and b.

*Solution 5* In both cases the MRS equals 1/2. They are the same as $u_2$ is as strictly increasing transformation of $u_1$.

*Solution 6* $\tilde{x}_1(p_1,p_2;m) = \frac{4m}{4p_1+3p_2}$ and $\tilde{x}_2(p_1,p_2;m) = \frac{3m}{4p_1+3p_2}$.

*Solution 7* a. $\sqrt{16}+2 > \sqrt{4}+3$.
b. $\sqrt{144}+18 < \sqrt{36}+27$.
c. No (by a and b).
d. $(1, \frac{43}{2})$.
e. If $m$ is large enough (such that $\tilde{x}_2 \geq 0$), then $\tilde{x}_1(p_1,p_2;m) = \frac{p_2^2}{4p_1^2}$, $\tilde{x}_1(p_1,p_2;m) = \frac{4p_1 m - p_2^2}{4p_1 p_2}$.

*Solution 8* a. 7/3.
b. $\tilde{x}_1 = 7\frac{p_2}{p_1} - 1$; $\tilde{x}_2 = \frac{m+p_1-7p_2}{p_2}$.
c. $(27,19)$ resp. $(10,0)$.

*Solution 9* $x_1 = 44,\ x_1{}' = 48.6$.
Short explanation: for the demand function of good 1 we have $\tilde{x}(p) = \frac{100-56p}{2-p^2}$.

*Solution 10* a. 7.
b. $\frac{df}{dx}\frac{x}{f(x)} = e^x\frac{x}{e^x} = x = 0$.

*Solution 11* Respectively $-34/25$, $-1/25$, $30/25$, $5/25$.

*Solution 12*

*Solution 13* a. Decreasing returns to scale.
b. Constant returns to scale.

*Solution 14* a. $(40^{1/3}, \frac{1}{4}40^{1/3}) = (4(\frac{10}{16})^{1/3}, (\frac{10}{16})^{1/3})$.
b. $(400/9, 100/9)$.
c. $(q/2, q/3)$, $c(q) = q(\frac{w_1}{2} + \frac{w_2}{3})$.

*Solution 15* a. The profit maximising production factor bundle is the solution of the two equations $w_1 = p\frac{\partial f}{\partial k_1}$ and $w_2 = p\frac{\partial f}{\partial k_2}$, i.e. of the equations

$$1 = 4 \cdot k_2^2 \text{ and } 2 = 4 \cdot 2k_1k_2.$$

Solving these equations gives $(k_1,k_2) = (1/2, 1/2)$. So the profit maximising output is $q = f(1/2,1/2) = 1/8$.

b. For the cost function $c(q)$ we first have to determine the conditional production factor functions. They are obtained by solving the two equations $w_1/w_2 = \frac{\partial f}{\partial k_1}/\frac{\partial f}{\partial k_2}$ and $f(k_1,k_2) = q$, i.e. of the equations

$$\frac{1}{2} = \frac{1}{2}\frac{k_2}{k_1} \text{ and } k_1k_2^2 = q.$$

Solving gives $\tilde{k}_1 = q^{1/3}$ and $\tilde{k}_2 = q^{1/3}$. This gives a cost function $c(q) = 3q^{1/3}$. Finally, price is marginal cost, i.e. $p = c'(q)$, gives for the profit maximising output $q = 1/8$. So the profit maximising production factor bundle is $k_1 = k_2 = 1/8^{1/3} = 1/2$.

*Solution 16* In the same way as in the previous exercise, but the algebra is more difficult.

a. The profit maximising production factor bundle is the solution of the two equations $w_1 = p\frac{\partial f}{\partial k_1}$ and $w_2 = p\frac{\partial f}{\partial k_2}$. Solving these equations gives $(k_1, k_2) = (16, 16)$. So the profit maximising output is $q = f(16, 16) = 8$.

b. For the cost function $c(q)$ we first have to determine the conditional production factor functions. They are obtained by solving the two equations $w_1/w_2 = \frac{\partial f}{\partial k_1}/\frac{\partial f}{\partial k_2}$ and $f(k_1, k_2) = q$. Solving gives $k_1 = q^{4/3}$ and $k_2 = q^{4/3}$. This gives a cost function $c(q) = 3q^{4/3}$. Finally, price is marginal cost, gives for the profit maximising output $q = 8$. So the profit maximising production factor bundle is $k_1 = k_2 = 8^{4/3} = 16$.

*Solution 17* The average variable cost function is $2q^2 - 16q + 64$. This function is minimal for $q = 4$; the minimum is 32. Thus the shutdown price is 32.

*Solution 18* a. Yes, due to the convexity of the utility function.

b. $\pi_1 c_1^2 + \cdots + \pi_n c_n^2$.

c. We have $\pi_1 = \frac{1}{4}, \pi_2 = \frac{3}{4}$.

'No' means gamble $g_0 = (100, 1)$.

$$U(g_0) = U((100, 1)) = 1 \cdot u(100) = 1 \cdot 100^2 = 10.000.$$

'Yes' means gamble $g_1 = ((120, \frac{1}{4}), (80, \frac{3}{4}))$.

$$U(g_1) = U((120, \frac{1}{4}), (80, \frac{3}{4})) = \frac{1}{4}120^2 + \frac{3}{4}80^2 = 8400,$$

As $U(g_1) < U(g_0)$, Joep will not participate.

*Solution 19* No insurance is gamble

$$g_0 := ((225, \frac{98}{100}), (25, \frac{2}{100})).$$

Insurance with price $x$ is gamble

$$g_1 := (225 - x, 1).$$

As $U(g_0) = 14,8$ and $U(g_1) = \sqrt{225 - x}$, $U(g_0) = U(g_1)$ gives $x = 5,96$.

*Solution 20* a. $\frac{1}{4}1000 + \frac{3}{4}100 = 325$ respectively $U((1000, 1/4), (100, 3/4)) = \frac{1}{4}U(1000) + \frac{3}{4}U(100)$.

b. Since he is risk-averse, he prefers the expected value of the gamble 325 to the gamble itself. Therefore he would take the payment.

*Solution 21* Marshallian demand functions are $\tilde{x}_1(p_1, p_2; m) = \frac{4m}{4p_1 + 3p_2}$ and $\tilde{x}_2(p_1, p_2; m) = \frac{3m}{4p_1 + 3p_2}$. So the indirect utility function is $v(p_1, p_2; m) = \frac{12m}{4p_1 + 3p_2}$.

*Solution 22* Marshallian demand functions are $\tilde{x}_1(p_1, p_2; m) = \tilde{x}_2(p_1, p_2; m) = \frac{m}{p_1 + p_2}$. So the indirect utility function is $v(p_1, p_2; m) = \frac{m}{p_1 + p_2}$. The formula for calculating the compensating variation $CV$ is $v(p_1, p_2; m) = v(p_1', p_2; m - \mathrm{CV})$, i.e. here $v(1, 3; 15) = v(4; 3, 15 - \mathrm{CV})$. Solving gives $\mathrm{CV} = -45/4$.

*Solution 23* $\mathrm{CV} = \mathrm{EV} = \Delta S = -1/2$.

Short solution:

$\tilde{x}_1 = \frac{p_2^2}{4p_1^2} = \frac{1}{p_1^2}$.

$\tilde{x}_2 = \frac{m - p_1\tilde{x}_1}{p_2} = \frac{m}{2} - \frac{1}{2p_1}$.

$v(p_1, m) = \sqrt{\tilde{x}_1} + \tilde{x}_2 = \frac{m}{2} + \frac{1}{2p_1}$.

Solving $v(1, 20) = v(2, 20 - \mathrm{CV})$ gives $\mathrm{CV} = -1/2$.

Solving $v(2, 20) = v(1, 20 + \mathrm{EV})$ gives $\mathrm{EV} = -1/2$.

$\Delta S = \int_2^1 \frac{1}{p_1^2} dp_1 = -1/2$.

*Solution 24* aT bF cF dT eT fT gF hT iF jF.

*Solution 25* aT bT cT dT eF fF gF hF iF jT.

*Solution 26* a. Strictly dominant strategy for player 1: 3, i.e. the third row.

Strictly dominant strategy for player 2: none.

Nash-equilibria: the strategy profile $(3, 2)$, i.e third row for player 1 and second column for player 2.

Fully cooperative strategy profiles: $(3, 2)$.

Pareto efficient strategy profiles: $(3, 1)$ en $(3, 2)$.

Prisoners' dilemma: no.

b. 2.

2.

$(2, 2)$.

$(1, 1)$.

$(1,1),(1,2),(2,1)$.
Yes.
c. None.
None.
$(1,1),(2,2)$.
$(1,1),(2,2)$.
$(1,1),(2,2)$.
No.
d. None.
None.
$(2,2)$.
$(2,2)$
$(1,2),(2,1),(2,2)$.
No.
e. None.
2.
$(3,2)$.
$(3,2)$.
$(1,2),(3,2)$.
No.
f. 2.
2.
$(2,2)$.
$(1,1),(1,2),(2,1)$.
$(1,1),(1,2),(2,1)$.
Yes.
g. None.
None.
$(3,1)$.
$(3,1)$.
$(1,2),(2,3),(3,1)$.
No.
h. None.
None.
$(1,3),(3,2)$.
$(1,3),(3,2)$.
$(1,3),(3,1),(3,2)$.
No.

*Solution 27* a. $\pi_1(q_1,q_2)=180q_1-\frac{1}{4}q_1^2-\frac{1}{4}q_1q_2,\ \pi_2(q_1,q_2)=190q_2--\frac{1}{4}q_2^2-\frac{1}{4}q_1q_2$.
b. $R_1(q_2)=360-\frac{1}{2}q_2,\ R_2(q_1)=380-\frac{1}{2}q_1$.
c. $q_1=680/3, q_2=800/3, p=230/3$.
d. $q_1=340, q_2=210, p=250/4$.
e. $q_1=0,\ q_2=380$.

*Solution 28*

*Solution 29* a. $\overline{x}_1^A=\frac{m^A}{2p_1}=\frac{5p1}{2p_1}=\frac{5}{2}$.
$\overline{x}_1^B=\frac{m^B}{p_1+p_2}=\frac{6p_2}{p_1+p_2}$.
b. $e_1^A=\overline{x}_1^A-\omega_1^A=\frac{5}{2}-5=-\frac{5}{2}$.
$e_1^B=\overline{x}_1^B-\omega_1^B=\frac{6p_2}{p_1+p_2}-0=\frac{6p_2}{p_1+p_2}$.
Thus $z_1=e_1^A+e_1^B=-\frac{5}{2}+\frac{6p_2}{p_1+p_2}$.
c. $z_1=0$ gives $p_2=5/7$.

*Solution 30* a. $\tilde{x}_1^A=\frac{m}{2p_1}-\frac{5}{2}+\frac{p_2}{p_1},\ \tilde{x}_2^A=\frac{m}{2p_2}+\frac{5}{2}\frac{p_1}{p_2}-1,\ \tilde{x}_1^B=\frac{m}{2p_1}-1+2\frac{p_2}{p_1},\ \tilde{x}_2^B=\frac{m}{2p_2}+\frac{p_1}{p_2}-2$.
b. $e_1^A=\frac{p_2}{p_1}-\frac{83}{2},\ e_2^A=\frac{83}{2}\frac{p_1}{p_2}-1,\ e_1^B=84\frac{p_2}{p_1}-1,\ e_2^B=\frac{p_1}{p_2}-84$.
c. $z_1=85\frac{p_2}{p_1}-\frac{85}{2},\ z_2=\frac{85}{2}\frac{p_1}{p_2}-85$.
d. Equilibrium prices: $\frac{p_2}{p_1}=\frac{1}{2}$.
Equilibrium quantities: $x_1^A=37,\ x_2^A=82,\ x_1^B=41,\ x_2^B=82$.
e. $MRS^A(37,82)=\frac{82+2}{37+5}=2$ and $MRS^B(41,82)=\frac{82+4}{41+2}=2$. Therefore pareto effcient.