WASS Course Game Theory

P. v. Mouche

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Exercise 1 *Given the following bi-matrix-game:*

$$\begin{pmatrix} 3;8 & 4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;4 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

Determine the nash equilibria.

Exercise 2 *Determine the equilibria for the homogeneous cournot oligopoly given by* $c^1(q^1) = 20q^1,\ c^2(q^2) = 20q^2,\ p(Q) = \max(200 - \frac{1}{4}Q, 0)$.

Exercise 3 *Consider the following game between* $n \geq 3$ *players. Each player* i *writes on a paper one of the numbers* $1, 2, \ldots, n$*, say* x^i*. Then a referee collects the papers and calculates for each player* i *the score by the formula*

$$-|x^i - \frac{1}{2}\frac{\sum_{l \in \mathcal{N}\setminus\{i\}} x^l}{n-1}|$$

(so minus the absolute value of the difference of his number minus half of the average of the numbers of the other players). Interpret this game in a natural way as a game in strategic form.

a. Show that this game is symmetric.

b. Is this game aggregative?

c. Are there dominant strategies?

d. For which $b \in \{1, \ldots, n\}$ *is the multi-strategy* $(b, b, \ldots, b)$ *a nash equilibrium?*

e. Optional exercise: determine all nash equilibria.

Exercise 4 *Consider the game in strategic form with two playes given by*

$$X^i = \mathbb{R}\ (i = 1, 2),\ f^i(x^1, x^2) = (100 - x^1 - x^2 - c)x^i\ (i = 1, 2)$$

where $c \in \mathbb{R}$

a. *Prove that this game is a symmetric aggregative game.*

b. *What do You find of the following way of Pietje Puk for determining the symmetric nash equilibria?*

Suppose $\mathbf{n}$ *is a symmetric nash equilibrium. Then* $n^1 = n^2 =: n$. *Maximising* $f^1(n, n) = (100 - 2n - c)n$ *gives* $n = (100 - c)/4$. *So* $((100 - c)/4, (100 - c)/4)$ *is a unique symmetric equilibrium..*

Short solutions.

Solution 1 They are $(1,1)$ (i.e. row 1 and column 1), $(1,2),(3,1),(3,2)$.

Solution 2 $(240,240)$.

Solution 3 a. $f^{\pi(j)}(x^1,\ldots,x^n) = -|x^{\pi(j)} - \frac{1}{2}\frac{\sum_{l\in\mathcal{N}\setminus\{\pi(j)\}} x^l}{n-1}| = -|x^{\pi(j)} - \frac{1}{2}\frac{\sum_{l\in\mathcal{N}\setminus\{j\}} x^{\pi(l)}}{n-1}| = f^j(x^{\pi(1)},\ldots,x^{\pi(n)})$.

b. Yes: $f^j(x^1,\ldots,x^n) = -|x^j - \frac{1}{2}\frac{\sum_{l\in\mathcal{N}\setminus\{j\}} x^l}{n-1}| = -|x^j - \frac{1}{2}\frac{-x^j+\sum_{l\in\mathcal{N}} x^l}{n-1}| = -|\frac{3}{2}x^j - \frac{1}{2}\frac{\sum_{l\in\mathcal{N}} x^l}{n-1}| = -|\frac{3}{2}x^j - \frac{1}{2}\frac{n}{n-1}\frac{\sum_{l\in\mathcal{N}} x^l}{n}|$.

c. No. This we can prove by looking for example to the best reply of i against $(b,\ldots,b)$. Well, $f^i(x^i;b,\ldots,b) = -|x^i - \frac{1}{2}b|$. Therefore $R^i(b,\ldots,b) = \{\frac{b}{2}\}$ if b is even, $R^i(b,\ldots,b) = \{\frac{b}{2}-1, \frac{b}{2}+1\}$ if b is odd and not equal to 1, and $R^i(b,\ldots,b) = \{b\}$ if $b=1$.

d. For $b=1$: the anwer to c implies that $R^i(1,\ldots,1) = 1$ $(i \in N)$ and if $a \neq 1$ that $R^i(a,\ldots,a) \neq a$ $(i \in N)$.

e. It can be proved that $E = \{(1,1,\ldots,1)\}$.

Solution 4 a. $f^2(x^1,x^2) = f^1(x^2,x^1)$ $(x_1,x_2 \in \mathbb{R})$.

b. This is not correct. In fact $((100-c)/3,(100-c)/3)$ is the unique nash equilibrium.