1. There are two goods, $x = (x_1, x_2) \in \mathbf{R}_+^2$. A consumer has the utility function $U(x) = u_1(x_1) + u_2(x_2)$ where each u_i is twice continuously differentiable with $u_i'(x_i) > 0$ and $u_i''(x_i) < 0$ for all $x_i \in \mathbf{R}_+^1$. Each u_i also satisfies the condition: $\lim_{x_i \to 0} u_i'(x_i) = +\infty$. Assume that prices of both goods are strictly positive, each $p_i > 0$, and wealth is strictly positive, $w > 0$.

   a.) Write the consumer's problem as a constrained optimization problem and display the first order conditions for this optimization problem.
   b.) Show that if wealth decreases then the demand for good 1 decreases.
   c.) What is the sign of the effect of a change in the price of good 1 on the consumer's demand for good 2? Show your work.

2. Provide a brief answer for each of the following questions.
   a.) A consumer has the expenditure function $e(p_1, p_2, u) = \dfrac{u p_1 p_2}{p_1 + p_2}$. Find this consumer's indirect utility function.
   b.) A firm has the profit function $\pi(p, w_1, w_2) = p^2 (w_1^{\alpha} + w_2^{\alpha})$, where p is the output price and w_1 and w_2 are input prices. Find this firm's output supply function.
   c.) A student has solved a cost minimization problem for a firm and concluded that the cost function is $c(w_1, w_2, y) = y w_1^{1/4} w_2^{1/4}$. Explain why his solution is incorrect.
   d.) Show that if each of the $i = 1, \ldots, I$ consumers in an economy has an indirect utility function of the form $v_i(p,w_i) = a_i(p) + b(p)w_i$, where $p \in \mathbf{R}_+^N$ is the price vector and $w_i \in \mathbf{R}_+^1$ is i's wealth, then aggregate demand can be written as a function of aggregate wealth $w = \sum_i^I w_i$.

3. An infinitely lived agent owns 1 unit of a commodity that he consumes over his lifetime. The commodity is perfectly storable and he will receive no more than he has now. Consumption of the commodity in period t is denoted x_t, and his lifetime utility function is given by

$$u(x_0, x_1, x_2, \ldots) = \sum_{t=0}^{\infty} \beta^t \ln x_t, \text{ where } 0 < \beta < 1.$$

   a.) Show that the solution to the consumer's utility maximization problem is unique.
   b.) Calculate consumer's optimal consumption level in each period. Provide intuition for this consumption pattern.
   c.) Now assume that the consumer will live only T years. Calculate consumer's optimal consumption level in each period in this case. How does the consumption pattern compare to your solution in b.)? Explain

4. Provide a brief answer to each of the following questions.

a.) Derive the <u>profit function</u> for the single-output technology whose production function is given by $f\left(\mathbf{z}\right)=\sqrt{z_1+z_2}$. The prices of inputs z_1 and z_2 are w_1 and w_2, respectively.

b.) Corn (C) is produced from labor (L) using a decreasing returns to scale technology of the

b.) form $C=AL^{\varepsilon}$, where A is a scale parameter and $\varepsilon\in\left(0,1\right)$. How is the parameter ε related to the price elasticity of the corn supply curve?

c.) Ethanol (E) is produced from corn (C) and labor (L) using a Leontief technology

$$E=\min\left(aC,bL\right),$$

where a and b are technological parameters. Draw the inverse ethanol supply curve and determine its price elasticity.

d.) When the ratio of goods consumed, x_i/x_j, is independent of income (m) for all i and j (i.e., $\partial\left(x_i/x_j\right)/\partial m=0$), then the ratio of any two income elasticities is always equal to 1 (i.e., $\varepsilon_i/\varepsilon_j=1$). True/false? Show your work.

5. A consumer has the expenditure function:

$$e\left(p,u\right)=up_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3},$$

where $\alpha_1+\alpha_2+\alpha_3=1,\ \alpha_i\geq 0$ for all i, u is a utility level, and the p_i are prices.

a.) Derive this consumer's indirect utility function.

b.) Derive the consumer's Marshallian demand functions for goods i = 1, 2, 3.

c.) Consider the following claim: The function:

$$z\left(x\right)=\alpha_1\ln\left(x_1\right)+\alpha_2\ln\left(x_2\right)+\alpha_3\ln\left(x_3\right)$$

generates the demands in part (b). Is this claim correct? Explain.

d.) Consider the following claim: The function:

$$z\left(x\right)=\alpha_1\ln\left(x_1\right)+\alpha_2\ln\left(x_2\right)+\alpha_3\ln\left(x_3\right),$$

represents this consumer's preferences. Is this claim correct? Explain.

6. Solve the following two problems

a.) Given the production function $f\left(x_{1,}x_{2,}\right)=\alpha_1\ln x_1+\alpha_2\ln x_2$, calculate the profit-maximizing demand and supply functions, and the profit function. For simplicity assume an interior solution. Assume that $\alpha_i>0$.

b.) A firm has two plants with cost functions $c_1\left(y_1\right)=4\sqrt{y_1}$ and $c_2\left(y_2\right)=2\sqrt{y_2}$. What is its cost of producing output y?