Exercises
UEC-51806 Advanced Microeconomics, Part 1
Instructor: Dr. Dušan Drabik, de Leeuwenborch 2105
Email: Dusan.Drabik@wur.nl

1. A consumer has a preference relation on $\mathbf{R}^1_+$ which can be represented by the utility function $u(x) = x^2 + 4x + 4$. Is this function quasi-concave? Briefly explain. Is there a concave utility function representing the consumer's preferences? If so, display one; if not, why not?

2. A consumer has Lexicographic preferences on $\mathbf{R}^2_+$ if the relationship $\succsim$ satisfies $\mathbf{x}^1 \succsim \mathbf{x}^2$ whenever $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_2^1 \geq x_2^2$. Show that lexicographic preferences on $\mathbf{R}^2_+$ are rational, i.e., complete and transitive.

3. A consumer with convex, monotonic preferences consumes non-negative amounts of x_1 and x_2.
   a.) If $u(x_1, x_2) = x_1^{\alpha} x_2^{\frac{1}{2}-\alpha}$ represents those preferences, what restrictions must there be on the value of parameter α? Explain
   b.) Given those restrictions, calculate the Marshallian demand functions.

4. In a two-good case, show that if one good is inferior, the other must be normal.

5. How would you determine whether the function
$$X(p_x, p_y, I) = \frac{2p_x I}{p_x^{\,2} + p_y^{\,2}}$$
could be demand function for commodity x of a utility maximizing consumer with preferences defined over the various combinations of x and y? Is it a demand function?

6. A firm produces output y from two inputs (x_1, x_2) using the production function $y = f(x_1, x_2)$. The output price is given by $p(y)$, the price of input one is w_1 per unit and the price of input two is w_2 per unit. That is, if the firm sells y units of output, the price it receives per unit is $p(y)$. Assume that f: $\mathbf{R}^2_+ \to \mathbf{R}^1_+$ is strictly concave and increasing and that p: $\mathbf{R}^1_+ \to \mathbf{R}^1_+$ is decreasing and convex. Both f and p are twice differentiable. Note that this firm is a price taker in the input market; its choices do not affect the input prices (w_1,w_2).

   a.) Write the firm's profit maximization problem and profit function. Let $\pi(w_1,w_2)$ be the profit function.
   b.) Is the partial derivative of $\pi(w_1,w_2)$ with respect to w_i equal to (−1) times the firm's input demand function for input i? Explain.
   c.) Is $\pi(w_1,w_2)$ a convex function of (w_1,w_2)? Explain.
   d.) Now suppose that $f(x_1, x_2) = x_1^{\beta} x_2^{1-\beta}$ and that $p(y) = y^{-\alpha}$, where $1 > \beta > 0$ and $1 > \alpha > 0$ Find the optimal input demands and output supply.

7. Consider a competitive firm with a well-behaved production function *f*(x) that converts input *x* into a product *q*. The market price of the product is *p*. Derive the relationship between the curvature of the production function, that is, f_{xx} and the elasticity of the product supply curve.

8. Given the production function $f\left(x_{1,} x_{2,}\right) = \alpha_1 \ln x_1 + \alpha_2 \ln x_2$, calculate the profit-maximizing demand and supply functions, and the profit function. For simplicity assume an interior solution. Assume that $\alpha_i > 0$.

9. Corn (C) is produced from labor (*L*) using a decreasing returns to scale technology of the form $C = AL^{\varepsilon}$, where *A* is a scale parameter and $\varepsilon \in (0,1)$. How is the parameter ε related to the price elasticity of the corn supply curve?