Exercises
UEC-51806 Advanced Microeconomics, Part 1
Instructor: Dr. Dušan Drabik, de Leeuwenborch 2105
Email: Dusan.Drabik@wur.nl

1. A consumer has a preference relation on $\mathbf{R}^1_+$ which can be represented by the utility function u(x) = $x^2$ + 4x + 4. Is this function quasi-concave? Briefly explain. Is there a concave utility function representing the consumer's preferences? If so, display one; if not, why not?

2. A consumer has Lexicographic preferences on $\mathbf{R}^2_+$ if the relationship $\succsim$ satisfies $\mathbf{x}^1 \succsim \mathbf{x}^2$ whenever $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_2^1 \geq x_2^2$. Show that lexicographic preferences on $\mathbf{R}^2_+$ are rational, i.e., complete and transitive.

3. A consumer with convex, monotonic preferences consumes non-negative amounts of $x_1$ and $x_2$.

   a.) If $u(x_1, x_2) = x_1^{\alpha} x_2^{\frac{1}{2}-\alpha}$ represents those preferences, what restrictions must there be on the value of parameter $\alpha$ ? Explain

   b.) Given those restrictions, calculate the Marshallian demand functions.

4. In a two-good case, show that if one good is inferior, the other must be normal.

5. How would you determine whether the function

$$X\left(p_x, p_y, I\right) = \frac{2 p_x I}{p_x^{\ 2} + p_y^{\ 2}}$$

   could be demand function for commodity x of a utility maximizing consumer with preferences defined over the various combinations of x and y? Is it a demand function?

6. A firm produces output *y* from two inputs ($x_1$, $x_2$) using the production function y = $f(x_1, x_2)$. The output price is given by $p(y)$, the price of input one is $w_1$ per unit and the price of input two is $w_2$ per unit. That is, if the firm sells *y* units of output, the price it receives per unit is $p(y)$. Assume that *f*: $\mathbf{R}^2_+ \rightarrow \mathbf{R}^1_+$ is strictly concave and increasing and that *p*: $\mathbf{R}^1_+ \rightarrow \mathbf{R}^1_+$ is decreasing and convex. Both *f* and *p* are twice differentiable. Note that this firm is a price taker in the input market; its choices do not affect the input prices ($w_1$,$w_2$).

   a.) Write the firm's profit maximization problem and profit function. Let $\pi(w_1,w_2)$ be the profit function.

   b.) Is the partial derivative of $\pi(w_1,w_2)$ with respect to $w_i$ equal to (−1) times the firm's input demand function for input *i*? Explain.

   c.) Is $\pi(w_1,w_2)$ a convex function of ($w_1$,$w_2$)? Explain.

   d.) Now suppose that $f\left(x_1, x_2\right) = x_1^{\beta} x_2^{1-\beta}$ and that $p\left(y\right) = y^{-\alpha}$, where $1 > \beta > 0$ and $1 > \alpha > 0$ Find the optimal input demands and output supply.

7. Consider a competitive firm with a well-behaved production function $f(x)$ that converts input $x$ into a product $q$. The market price of the product is $p$. Derive the relationship between the curvature of the production function, that is, $f_{xx}$ and the elasticity of the product supply curve.

8. Given the production function $f(x_{1,} x_{2,}) = \alpha_1 \ln x_1 + \alpha_2 \ln x_2$, calculate the profit-maximizing demand and supply functions, and the profit function. For simplicity assume an interior solution. Assume that $\alpha_i > 0$.

9. Corn (C) is produced from labor ($L$) using a decreasing returns to scale technology of the form $C = AL^{\varepsilon}$, where $A$ is a scale parameter and $\varepsilon \in (0,1)$. How is the parameter $\varepsilon$ related to the price elasticity of the corn supply curve?

10.

*Consider a pure exchange with two consumers. Both consumers have Cobb-Douglas preferences, but with different parameters. Consumer 1 has utility function $u(x_1, y_1) = x_1^{\alpha} y_1^{1-\alpha}$. Consumer 2 has utility function $u(x_2, y_2) = x_2^{\beta} y_2^{1-\beta}$. The endowment of good $x$ ($y$) owned by consumer $i$ is $\bar{x}_i$ ($\bar{y}_i$). The price of good 1 is $p$, while the price of good 2 is normalized to 1 without loss of generality.*

(a) *Only for part (a), assume $\bar{x}_1 = 1$, $\bar{y}_1 = 3$, $\bar{x}_2 = 3$, $\bar{y}_2 = 1$. (That is, total endowment of each good is 4). Assume further $\alpha = 1/2$, $\beta = 1/2$. Draw the Pareto set and the contract curve for this economy in an Edgeworth box. (You do not need to give the exact solutions, only a graphical representation.) What is the set of points that could be the outcome under barter in this economy?*

(b) *For each consumer, compute the utility maximization problem. Solve for $(x_i, y_i)$ for $i = 1, 2$ as a function of the price $p$ and of the endowments.*

(c) *Now comes the general equilibrium part. Require now that the total sum of the demands for good $x$ equals the total sum of the endowments, that is, that $x_1 + x_2 = \bar{x}_1 + \bar{x}_2$. Solve for the general equilibrium price $p^*$.*

(d) *What is the comparative statics of $p^*$ with respect to the endowment of good $x$, that is, with respect to $\bar{x}_i$ for $i = 1, 2$? What about with respect to the endowment of the other good? Does this make sense? What is the comparative statics of $p^*$ with respect to the taste for good 1, that is, with respect to $\alpha$ and $\beta$? Does this make sense?*