Microeconomics: Consumer's Surplus

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Welfare measures

Question: How can we value the benefit for consumers (producers) from consumption (production) and changes therein?

- `https://www.youtube.com/watch?v=KeMjSQYTSUk`

Compensating and equivalent variation (ctd.)

- Setting: consider a change of prices and budget. For simplicity we only consider a change of the price of good 1. The price is p_1 and becomes p'_1.
- This leads for the consumer to a utility change (new utility minus old utility).
- The compensating variation CV is the amount of money that when taken away from the consumer after the price change cancels the utility change.
- The equivalent variation EV is the amount of money that when given to the consumer before the change gives the same utility effect as the the price change.
- Different definitions of these notions exist in the literature concerning ”taken away” and “given to”; do not worry!

Indirect utility function

Given a consumer with an utility function

$$u(x_1, x_2),$$

we can calculate the maximum utility the consumer can reach given prices p_1, p_2 and income m. This utility level is denoted by

$$v(p_1, p_2; m).$$

The function v is called the 'indirect utility function'.
So the indirect utility function gives the maximal utility the consumer can reach given the prices and income.

Indirect utility function (ctd)

Example:

$$u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2},$$

Optimal $x_1 = \frac{\alpha_1}{\alpha_1+\alpha_2}\frac{m}{p_1}$.
Optimal $x_2 = \frac{\alpha_2}{\alpha_1+\alpha_2}\frac{m}{p_2}$.
Thus

$$v(p_1, p_2; m) = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\frac{m}{p_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\frac{m}{p_2}\right)^{\alpha_2}.$$

Compensating and equivalent variation (ctd.)

- CV and EV have the same sign as the utility change but need not to be equal.
- With v the indirect utility function (suppressing p_2 in the notation)

$$v(p_1; m) = v(p_1'; m - \mathrm{CV}),$$

$$v(p_1; m + \mathrm{EV}) = v(p_1', m).$$

- Because utility is not observable, it is a problem to determine CV and equivalent variation EV.

Consumer's surplus

- Setting: consider a change of the price of good 1. The price is p and becomes p'. Demand function for good 1 is D.
- (Net) consumer's surplus at price p_1 is area above price under demand curve.
- Change in consumer's surplus equals $\Delta S = \int_{p'}^{p} D(p)\, dp$.
- Consumer's surplus is observable as demand is observable.

Example

Assume we have a demand curve characterized by $D(p) = 20 - p^2$. The price changes from $p = 2$ to $p' = 3$. What is the change in consumer's surplus ΔS?

Answer: $\Delta S = \int_{p'}^{p} D(p) dp = \int_{3}^{2} (20 - p^2)\, dp$.
As an antiderivative of $20 - p^2$ is $20p - \frac{1}{3}p^3$, we obtain

$$(20 \cdot 2 - \frac{1}{3}2^3) - (20 \cdot 3 - \frac{1}{3}3^3) = 37\frac{1}{3} - 51 = -13\frac{2}{3}.$$

Example

$$u(x_1, x_2) = \min(x_1, x_2),$$

$$p_1 = 1,\ p_1' = 2,\ m = 12,\ p_2 = 2.$$

- $v(P, p_2; M) = \frac{M}{P+p_2}$. So $v(P; M) = \frac{M}{P+2}$.
- $v(1; 12) = v(2; 12 - \mathrm{CV})$ becomes $\frac{12}{3} = \frac{12-\mathrm{CV}}{4}$. Thus $\mathrm{CV} = -4$.
- $v(1; 12 + \mathrm{EV}) = v(2; 12)$ becomes $\frac{12+\mathrm{EV}}{3} = \frac{12}{4}$. Thus $\mathrm{EV} = -3$.
- $D(p) = \frac{m}{p+p_2} = \frac{12}{p+2}$. So $\int_{p'}^{p} D(p)\,dp = \int_2^1 \frac{12}{p+2}dp = 12(\ln(3) - \ln(4)) = -3.452...$

CV, EV, ΔS compared

- Under mild conditions, ΔS is an ‘average’ of CV and EV.
- If good 1 is quasi-linear, then under mild conditions $\mathrm{CV} = \mathrm{EV} = \Delta S$ holds.
- Summing the consumer’s surpli of the consumers leads to the consumers’ surplus.

Producer's surplus

- Setting: only price of output good changes.
- Producer's surplus at price p equals area under price p above supply curve.
- Summing the producer's surpli of the producers leads to the producers' surplus.

Cost benefit analysis: calculation of costs and benefits of various economic policies happens in practice often by calculating the effect of the policy on the total surplus, i.e. on the sum of the consumers' and producers' surplus.