# Equilibrium Uniqueness Results for Cournot Oligopolies Revisited
# CORRECTIONS AND SUPPLEMENTS

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Corrections:

1. *On Page 212, line 4* $\uparrow$*:* ... that $Df_i(\mathbf{z})(x_i) = \cdots$
2. *On Page 212, line 2* $\uparrow$*:* ... obtain $Df_i^{(\mathbf{z})}(a_i) = t_i(a_i, a_i + \underline{\mathbf{z}}) \geq t_i(a_i, b_i + \underline{\mathbf{z}}) \geq t_i(b_i, b_i + \underline{\mathbf{z}}) = \cdots$
3. *On Page 213, repace Proposition 3 (with its footnote) and its proof by the following*:

   *Proposition 3.* Sufficient for all conditional payoff functions of player $i$ to be strictly quasi-concave and strictly pseudo-concave on $\mathrm{Int}(X_i)$ is that

   – there exists a full marginal reduction $(t_i; q)$ of $f_i$;

   – $t_i$ is differentiable on $\mathrm{Int}(X_i) \times \mathrm{Int}(Y_q)$ and $D_1 t_i, D_2 t_i : \mathrm{Int}(X_i) \times \mathrm{Int}(Y_q) \to \mathbb{R}$ are continuous;

   – for all $x_i \in \mathrm{Int}(X_i)$ and $y \in \mathrm{Int}(Y_q)$ with $q_i x_i \leq y$

   $$t_i(x_i, y) = 0 \ \Rightarrow \ (D_1 t_i + q_i D_2) t_i(x_i, y) < 0. \ \diamond$$

   *Proof.* Fix $\mathbf{z} \in \mathbf{X}_{\hat{\imath}}$. Write $a = \sum_l q_l z_l$. Consider $h = f_i^{(\mathbf{z})} \upharpoonright \mathrm{Int}(X_i)$. We have $Dh(x_i) = Df_i^{(\mathbf{z})}(x_i) = t_i(x_i, q_i x_i + a)$ and $D^2 h(x_i) = D_1 t_i(x_i, q_i x_i + a) + q_i D_2 t_i(x_i, q_i x_i + a)$. So $h$ is twice continuously differentiable function. For all $x_i \in \mathrm{Int}(X_i)$ we have $Dh(x_i) = 0 \Rightarrow D^2 h(x_i) < 0$. Théorème 9.2.6. in Truchon (1987) guarantees that $h$ is, as desired, strictly pseudo-concave. So $h$ is strictly quasi-concave. As $f_i^{(\mathbf{z})}$ is continuous, it follows that also $f_i^{(\mathbf{z})}$ is strictly quasi-concave. $\square$
4. *Page 213, line 6* $\uparrow$*:* ... sufficient for the existence of an equilibrium $\mathbf{e}$ with $e_i \in W_i$ $(i \in N)$:
5. *On Page 216, lines 15, 16* $\downarrow$*:* we obtain $\tilde{f}_i(x_i; \mathbf{e}_{\hat{\imath}}) = \tilde{p}(x_i + a)x_i - c_i(x_i) \leq \tilde{p}(e_i + a)e_i - c_i(e_i) = \tilde{f}_i(e_i; \mathbf{e}_{\hat{\imath}})$.

6. *On Page 214, line 9 ↓:* ...But, by (2)-(4) the contradiction ...

7. *On Page 216, line 16 ↑:* ...and $c_i \restriction X_i \cap [0, v]$. Theorem 2 ...

8. *Page 218, line 2 ↓:* $X_i = \mathbb{R}_+$ the weak ...

9. *Page 218, line 3 ↓:* ...equivalent. With Proposition 11(3) we see that the marginal ...

10. *Page 218, line 13 ↓:* ...In case $X_i = \mathbb{R}_+$, (9) ...

11. *Proposition 11 (4) should be:* In case $X_i = \mathbb{R}_+$, (9) implies (11).

12. *Page 218, line 16 ↑:* that $Dp(y) \leq 0$. As $p$ is twice differentiable now also $Dp(0) \leq 0$ follows.

13. *Page 220, line 13 ↑:* strictly quasi-concave and on $\mathrm{Int}(X_i)$ strictly pseudo-concave.

14. *Replace Proposition 17 and its proof by:*

    **Proposition 17.** Fix $i \in N$. Suppose $c_i$ is increasing, $p$ has a non-zero market satiation point $v$ and $p(y) = 0$ for all $y \in Y$ with $y \geq v$. Also suppose $p$ is continuous, decreasing and $p \restriction [0, v[$ is log-concave and twice continuously differentiable. Suppose $c_i$ is twice continuously differentiable on $X_i \cap [0, v[$ and continuous at $v$ if $v \in X_i$. Finally suppose for all $y \in [0, v[$ and $x_i \in X_i$ with $x_i \leq y$

    $$Dp(y) - D^2 c_i(x_i) < 0.$$

    Then each conditional profit function of firm $i$ is quasi-concave. And each conditional profit function $f_i^{(\mathbf{z})}$ with $\underline{\mathbf{z}} < v$ is strictly quasi-concave and is on $\mathrm{Int}(X_i)$ strictly pseudo-concave. ⋄

    *p*roof. Fix $\mathbf{z}$ with $\underline{\mathbf{z}} < v$. Let $I = X_i \cap [0, v - \underline{\mathbf{z}}[$ and $h = f_i^{(\mathbf{z})} \restriction \mathrm{Int}(I)$. The function $h$ is twice continuously differentiable. With $y = x_i + \underline{\mathbf{z}}$ we have $Dh(x_i) = Dp(y)x_i + p(y) - Dc_i(x_i)$ and $D^2 h(x_i) = 2Dp(y) + D^2 p(y) x_i - D^2 c_i(x_i)$. As in the proof of Proposition 3 it follows for each $x_i \in \mathrm{Int}(I)$ that $Dh(x_i) = 0 \Rightarrow D^2 h(x_i) < 0$. Again it follows that $h$ is strictly pseudo-concave on $\mathrm{Int}(X_i)$ and therefore strictly quasi-concave on $\mathrm{Int}(X_i)$. As $f_i^{(\mathbf{z})}$ is continuous, it follows that also $f_i^{(\mathbf{z})}$ is strictly quasi-concave. Finally, Proposition 13 implies that each conditional profit function is quasi-concave. □

15. *On Page 221, delete Proposition 18 and the line above it.*

16. *Page 221, line 13 ↓:* ...that the following is true: ...

17. *Page 221, line 14 ↓:* ...$c_i$ is convex and increasing, then ...

18. *Page 221, line 8 ↑:* ...maximiser of $(r_{i;\underline{\mathbf{z}}} - c_i) \restriction W_i$ and

19. *Page 224, line 19 ↑:* is increasing ...

20. *Page 225, line 11 ↓:* ...and $p \restriction [0, v]$ is twice ...

21. *On Page 225, add the following condition to Theorem 9:*

i. for every $i$ and $\mathbf{z} \in \mathbb{R}^{n-1}$ with $\underline{\mathbf{z}} \in [0, v[$ the conditional profit function $f_i^{(\mathbf{z})}$ is strictly pseudo-concave on $[0, v - \underline{\mathbf{z}}[$. ...

Comments:

Further reading:

P. v. Mouche and F. Quartieri. On the Uniqueness of Cournot Equilibrium in Case of Concave Integrated Price Flexibility. Journal of Global Optimization: DOI 10.1007/s10898-012-9926-z.

If you think that some other things should be added here, then please let me know.