Producer theory

a. Provide for any cost function differentiable in y a proof of the following two statements: 1) As long as the marginal cost exceeds average cost, the average cost increases. 2) As long as the marginal cost equals the average cost, the average cost is constant.

b. Given the cost function $c(\mathbf{w}, y) = y(\beta_1 w_1 + \beta_2 w_2)$, determine the conditional input demand functions for $x_1(\mathbf{w}, y)$ and $x_2(\mathbf{w}, y)$. Give an interpretation of your result with respect to the parameters y and $\mathbf{w}$. Could you think of any technology which might correspond to this cost function?

c. Assume all inputs belong to either subgroup S or K. Check whether the following production functions are strongly, weakly or not separable at all:

$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} x_i x_j, \tag{1}$$

where $x_i \in S$

$$y = A x_1^{\alpha_1} x_2^{\alpha_2} + B x_3^{\alpha_3} x_4^{\alpha_4}, \tag{2}$$

where $x_1, x_2 \in S;\ x_3, x_4 \in K$

$$y = A \left[(\alpha_1 x_1^{\rho_1} + \alpha_2 x_2^{\rho_1})^{\rho_2/\rho_1} + \alpha_3 x_3^{\rho_2} \right]^{1/\rho_2}, \tag{3}$$

where $x_1 \in S; x_2 \in K; x_3 \notin S \cup K$

a. Anton has the utility function, $U_A = Ax_1^\alpha x_2^\beta$, and Brigitte has the utility function, $U_B = Bx_1^\gamma x_2^\eta$, where $\alpha, \beta, \gamma, \eta, A$, and B are positive and constant. Suppose $A \neq B, \alpha \neq \gamma$ and $\beta \neq \eta$. Is it possible that Anton and Brigitte have the same preferences? If yes, explain under what condition. Support your answer mathematically.

b. Consider a consumer with compensated demand functions of the form $x_1^h = 1 + 3p_1^{-0.5}p_2^a$ and $x_2^h = 1 + bp_1^{0.5}p_2^c$. Utility has been set equal to 1 for convenience. What are the values of the parameters a, b, and c and why?

c. For the following expenditure function determine if it is increasing and unbounded above in u, increasing in prices and homogeneous of degree one in prices: $e(\mathbf{p}, u) = u^{1/2}(2p_1^{1/2}p_2^{1/2})$.

d. A money metric utility function is defined as $m(\mathbf{p}, \mathbf{x}) \equiv e(\mathbf{p}, u(\mathbf{x}))$, where $\mathbf{p}$ is the vector of prices, $\mathbf{x}$ is the vector of quantities, e represents the expenditure function and u is the utility function. The money metric utility function can be interpreted as showing the amount of money m a consumer would need at prices p to be as well off as he could be by consuming the bundle of goods $\mathbf{x}$. Derive the money metric utility function of the consumer with the expenditure function given in c.

Solution to *Producer theory* a. Calculate marginal cost, average cost and slope of average cost for

$$c(w,y): MC = \frac{\partial c}{\partial y};\ AC = c(w,y)/y;\ \partial AC/\partial y = \left(\frac{\partial c}{\partial y} * y - c(w,y)\right)/y^2 = (MC{-}AC)/y$$

. As long as $MC > AC$ $\frac{\partial AC}{\partial y} > 0$ Average cost is increasing; as long as $MC = AC$ $\frac{\partial AC}{\partial y} = 0$ Average cost is constant.

b. Use Shephard's lemma to derive the conditional input demand functions: $\partial c/\partial w_i = x_i = y\beta_i$. Thus, the number of units of each input used is constant no matter what the input prices are. The input demand functions are increasing in output given $\beta_1, \beta_2 > 0$. The cost function corresponds to the fixed proportions Leontief technology $y = \min \frac{x_1}{\beta_1}, \frac{x_2}{\beta_2}$, where $\beta_1, \beta_2 > 0$.

c. 1 is not separable at all; 2 weakly separable in S and K; 3 strictly separable.

Solution to *Consumer theory* a. both have the same type of preferences; MRS are equal when ratio of exponents are equal: $\alpha/\beta = \gamma/\eta$

b. compensated demand functions should be homogeneous of degree zero in prices $\rightarrow$ sum of the exponents should be zero: $a = 1/2, c = -1/2$; substitution matrix should be symmetric (alternatively, Young's theorem states that second order partial derivative of expenditure function should be the same) $\rightarrow \partial x_1^h/\partial p_2 = \partial x_2^h/\partial p_1$: $b = 6a = 3$

c. increasing in u (first partial derivative with respect to $u > 0$); unbounded above: $\sqrt{p_1 p_2}/\sqrt{u}$ unlimited; increasing in prices, homogeneous of degree one in prices

d. derive the compensated demand functions $x_i^h = u^{1/2} p_i^{-1/2} p_j^{1/2}$, multiply and solve for utility: $u = x_1^h x_2^h p_1^0 p_2^0 = x_1 x_2$, substitute into expenditure function $e = (x_1 x_2)^{1/2} 2 p_1^{1/2} p_2^{1/2}$