1.

There are two goods, $x = (x_1, x_2) \in \mathbf{R}^2_+$. A consumer has the utility function $U(x) = u_1(x_1) + u_2(x_2)$ where each u_i is twice continuously differentiable with $u_i'(x_i) > 0$ and $u_i''(x_i) < 0$ for all $x_i \in \mathbf{R}^1_+$. Each u_i also satisfies the condition: $\lim_{x_i \to 0} u_i'(x_i) = +\infty$. Assume that prices of both goods are positive, that is, $p_i > 0$, and wealth is positive, that is, $w > 0$.

a.) Write the consumer's problem as a constrained optimization problem and display the first order conditions for this optimization problem. (*2 points*)

b.) Show that if wealth decreases then the demand for good 1 decreases. (*4 points*)

c.) What is the sign of the effect of a change in the price of good 1 on the consumer's demand for good 2? Show your work. (*4 points*)

2. Provide a brief answer for each of the following questions.

a.) A consumer has the expenditure function $e(p_1, p_2, u) = \dfrac{u p_1 p_2}{p_1 + p_2}$. Find this consumer's indirect utility function. (*3 points*)

b.) A firm has the profit function $\pi(p, w_1, w_2) = p^2 (w_1^{\alpha} + w_2^{\alpha})$, where p is the output price and w_1;w_2 are input prices. Find this firm's output supply function. (*3 points*)

c.) A student has solved a cost minimization problem for a firm and concluded that the cost function is $c(w_1, w_2, y) = y w_1^{1/4} w_2^{1/4}$. Explain why his solution is incorrect. (*1 point*)

d.) Show that if each of the $i = 1, \ldots, I$ consumers in an economy has an indirect utility function of the form $v_i(p,w_i) = a_i(p) + b(p)w_i$, where $p \in \mathbf{R}^N_+$ is the price vector and $w_i \in \mathbf{R}^1_+$ is i's wealth, then aggregate demand can be written as a function of aggregate wealth $w = \sum_i^I w_i$. (*3 points*)

3. A contest game *with complete (part a-c) and incomplete information (d-g).*

There are two agents $i = 1, 2$ competing for a prize P that has subjective valuation $v_i(P)$. Both agents invest simultaneously to obtain the prize. The cost of the investment is either 1 or 2, i.e. $c_i \in \{1, 2\}$.

Agent *i*'s probability of winning the prize is $\frac{c_i}{c_1 + c_2}$.

a) Determine the payoff function. (*1 point*)

b) Draw the game tree that represents the game as an extensive form game. (*1 point*)

c) Show that high investments ($c_i = 2$) are a (strictly) dominant strategy if the valuation of the prize $v_i(P) > 6$. (*2 points*)

Now consider incomplete information. Suppose the agents either place a high value on the prize $v_i^H(P) = 9$ or a low value $v_i^L(P) = 3$. The common prior probabilities are given in the following table:

	v_2^L	v_2^H
v_1^L	0.16	0.24
v_1^H	0.24	0.36

d) Draw the game tree that reflects that players' types are randomly determined. (*2 points*)

e) Calculate the expected payoff of a player with a high valuation for each of his investment strategies. (2 *point*)

f) Calculate the expected payoff of a player with a low valuation for each of his investment strategies. (*1 point*)

g) Describe the Bayesian-Nash Equilibria of the game. (*1 point*)

4. Expected Utility

One definition of risk aversion is this:

An agent is risk averse if $u(E(g)) > u(g)$,

where g is a gamble, u is a vNM-utility function and $E(g)$ is the expected outcome of gamble g.

a) Show that an agent with $u(w) = \alpha \ln w$ ($\alpha > 0$) is risk averse according to the definition when faced with the gamble $g = (\frac{1}{2} \circ (w_0 + h), \frac{1}{2}(w_0 - h))$. (3 *points*)

[It may be helpful to know that $\ln x + \ln y = \ln(xy)$ and $y \cdot \ln x = \ln(x^y)$]

b) Calculate the certainty equivalent, $\hat{c}$. (3 *points*)

c) Recall the definition of a risk premium: $p = E(g) - \hat{c}$. Is the risk premium associated with gamble g increasing or decreasing in initial wealth? Interpret your finding. (2 *points*)

d) The variable h signifies a mean preserving spread. Argue that an increase in h means a larger risk. (2 *points*)