Advanced Microeconomics(ECH 32306)

*Homework 3 --- **please submit before Tuesday, 7 October 2014, 10.30 h***

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Expected Utility Theory

1. On p.100 Jehle and Reny say that AXIOM G4 (Monotonicity) implies $a_1 \succ a_n$. Prove this.

2. Which of the following utility functions have the DARA property, that is, the Arrow-Pratt measure of absolute risk aversion R_a is decreasing in wealth.

(i) $u(w) = \ln w$
(ii) $u(w) = -a(w^2 - 2bw)$; $a, b > 0$
(iii) $u(w) = 1 - e^{1-w}$
(iv) $u(w) = aw^\alpha$; $a > 0, \alpha > 0$

(v) Discuss why or why not DARA is a convincing property.

3. A sports fan has an expected utility function $u(w) = \ln w$. She has subjective probability p that her favourite club "Ajax" will win the next match and probability $1-p$ that they will not win. She chooses to bet €*x* on Ajax such that if Ajax wins, she wins €*x*, but if Ajax does not win she loses €*x*. The fan's initial wealth is w_0.

(a) Determine the fan's degree of absolute and relative risk aversion.
(b) How can you determine the fan's subjective odds $p/(1-p)$ by observing the size of his bet?
(c) How much would she bet if $p = \frac{3}{5}$ and $w_0 = 9$?

Now suppose for the next game the fan has received a betting ticket from a friend as gift. If Ajax wins, she receives €2*x*, but if Ajax does not win, her loss is already paid for. This time $p = 1/2$.

(d) For how much would the fan sell her betting ticket depending on *x* and w_0?
(e) Calculate the subjective money value of the gift when $w_0 = 9$ and $x = 1$.

Value of Information

4. Consider a chemical. Its use gives a benefit

$$B(x) = x^\beta,$$

where *x* is the level of exposure, β ($0 < \beta < 1$) is a substance specific parameter. In addition the use of the substance gives a damage $D(x, \tau) = x\tau$ that depends on exposure and the substance's toxic potential τ. Suppose, through adequate measures the regulator can control exposure.

(a) How should the regulator set exposure of a substance with known toxic potential?
(b) Now suppose the toxic potential is unknown but known to be either low τ_l or high τ_h with probabilities p and $1-p$, respectively. How should exposure be regulated?
(c) Consider case (b) and assume $\tau_l = 0.1$, $\tau_h = 10$, $\beta = 0.5$, and $p = 2/3$. Calculate optimal exposure and the expected welfare (benefits minus damage cost)
(d) Using the specification in (c), what is the regulators willingness to pay for a test that provides perfect information about the substance's toxic potential?

5. Let $A = \{a_1, a_2, a_3\}$ where $a_1 \succ a_2 \succ a_3$. A gamble *g* offers a_2 with certainty. Prove that if $a_2 \sim (\alpha \circ a_1, (1-\alpha) \circ a_3)$, then α must be strictly between zero and one.

Games with incomplete Information

6. *The ultimatum game*. Consider the following two-player game. Player 1 receives *x* coins of 1€ and *x* is a strictly positive even number. Player 1 offers none or a positive number of these coins to player 2. If player 2 rejects the offer, all the money is withdrawn. If player 2 accepts the offer, she receives what is offered to her and player 1 keeps the rest.

(a) Describe in a formal way the strategy spaces of the players and draw a game tree.

(b) What type of game is this and what is an appropriate solution concept?

In experiments many people reject strictly positive offers in the ultimatum game if these are considered "unfair". Suppose $\frac{1}{3}$ of the population has strong fairness preferences such that if they were in the role of player 2, they would reject any offer which is worth less than $\frac{1}{2}x$.

(c) What type of game is this and what is an appropriate solution concept? Draw the game tree (Hint: Harsanyi introduced a method to represent this kind a game).

(d) Derive the equilibrium.

(e) Determine the threshold fraction of "fair" players in the population such that only "fair" offers occur in equilibrium.

The ultimatum game is a famous game from the early days of experimental economics (W Güth, R Schmittberger, B Schwarze (1982) An Experimental Analysis of Ultimatum Bargaining. Journal of Economic Behavior and Organisation 3, 367-388.)

7. *A principle-agent game*: There are two agents: a risk neutral land owner and a risk averse farmer. Harvest is subject to risk. The risk is impacted be the farmer's effort. The land owner typically cannot observe the farmer's effort. Only harvest is observable.

a) Argue why or why not the land owner should pay a fixed wage to the farmer.

Assume now that effort *e* is either low $e_l = 0$ or high $e_h = 1$. With low effort harvest is either low $x_l = 15$ with probability $\frac{2}{3}$ or high $x_h = 60$ with probability $\frac{1}{3}$. With high effort harvest is low with probability $\frac{1}{3}$ or high with probability $\frac{2}{3}$. Furthermore assume the farmer can earn an off-farm wage $w_0 = 30.25$ with effort $e_0 = 0.5$. The landowner maximises profits (assume the price of the crop is 1). The farmer's utility function is $u(w,e) = \sqrt{w} - e$.

b) Give a formal description of the land owner's maximisation problem. What are the relevant constraints the land owner faces when offering a contract to the farmer? Describe the contract that the land owner offers to the farmer.

c) Will this contract be accepted? Calculate the land owner's profit if the contract is accepted.

8. *Market for lemons*. Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterised by the quality of the used car he wishes to sell. Let $\theta \in [0,1]$ index the quality of a used car and assume that θ is uniformly distributed on [0, 1]. If a seller of type θ sells his car (of quality θ) for a price *p*, his utility is $u_s(p,\theta)$. If he does not sell his car, then his utility is 0. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price *p* and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential *k* buyers.

a) Argue that in a competitive equilibrium under asymmetric information, we must have $E(\theta \mid p) = p$.
b) Show that if $u_s(p,\theta) = p - \frac{\theta}{2}$, then every $p \in [0, \frac{1}{2}]$ is an equilibrium price.
c) Find the equilibrium price when $u_s(p,\theta) = p - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?
d) Find an equilibrium price when $u_s(p,\theta) = p - \theta^3$. How many equilibria are there in this case?