Advanced Microeconomics: Part 3

Risk, uncertainty and information

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Overview

a. Expected Utility Theory (EUT)
b. Value of information
c. Games with incomplete information
    - solution concepts
    - insurance (adverse selection)
    - the principal-agent problem (moral hazard)

Make a choice

$g_1 = 1000$ for sure

or

$$g_2 = \begin{cases} \text{with} & 0.1 \text{ chance} & 5000 \\ \text{with} & 0.89 \text{ chance} & 1000 \\ \text{with} & 0.01 \text{ chance} & 0 \end{cases}$$

Make another choice

$$g_3 = \begin{cases} \text{with} & 0.1 \text{ chance} & 5000 \\ \text{with} & 0.9 \text{ chance} & 0 \end{cases}$$

or

$$g_4 = \begin{cases} \text{with} & 0.11 \text{ chance} & 1000 \\ \text{with} & 0.89 \text{ chance} & 0 \end{cases}$$

Literature

Textbook

Jehle and Reny, section 2.4.

Seminal works

Bernoulli, Daniel (1738 / 1954) Exposition of a new theory on the measurement of risk. (Translation by Louise Sommer from Latin: Specimen Theoriae Novae de Mensura Sortis. In: Commentarii Academiae Scientiarum Imperialis Petropolitanae V, 1738, 175-192.) Econometrica 22(1), 23-36.

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Additional Reading

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Basic Concepts (1)

- Outcomes: $A = \{a_1, a_2, \ldots, a_n\}$
- Probabilities: Each outcome a_i occurs with probability p_i
- Simple gambles: $g = (p_1 \circ a_1, \ldots, p_n \circ a_n)$
  such that $\sum_i p_i = 1$
- The set of simple gambles is
  $\mathcal{G}_s = \{(p_1 \circ a_1, \ldots, p_n \circ a_n) | p_i \geq 0, \sum_i p_i = 1\}$.
- If a lottery has another lottery as its prize, we have to deal with compound gambles.

Preferences are an ordering "$\succeq$" over the set of gambles.

Expected Utility Theory (EUT)

Basic Concepts (2)

- Certainty: state of the world is known for very action
- Risk: known set of states of the world and probabilities associated with actions
- Uncertainty: known set of states of the world but unknown probabilities
- Deep uncertainty: Unknown set of states.

The distinction between risk and uncertainty is due to Frank H. Knight (1920).
EUT deals with risk.

EUT: Example

A bank decides whether to give a mortgage to a customer or not.
Actions: $A =$ {give no loan, give a loan}
States: $S =$ {good business, no business, bad business}
Probabilities: (p_1, p_2, p_3)

Actions are choices between gambles.
Two simple gambles: giving the loan and not giving the loan.

$g_1 = (0 \circ \text{payback with interest}, 1 \circ \text{no loan}, 0 \circ \text{bad debt})$
$g_2 = (p_1 \circ \text{payback with interest}, 0 \circ \text{no loan}, (1 - p_1) \circ \text{bad debt})$

How to choose? A possible decision criterion is: maximise expected payoffs.
But . . . (see next slide)

EUT: St. Petersburg Paradox

Daniel Bernoulli (1700-1782)

EUT: St. Petersburg Paradox

The casino in St. Petersburg offers the following gamble:
You toss a coin until heads comes up. If heads comes up at the nth toss, you receive a payoff of 2^n roubles.
How much would you offer to participate in the gamble?

n	1	2	3	...
payoff	2	4	8	...
probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	...
expected payoff	1	1	1	...

The expected value of the lottery is infinity!
Daniel Bernoulli was the first to draw the important distinction between a money payoff and its (subjective) worth.
The St. Petersburg Paradox is resolved if value (utility) is a concave function of money payoff.

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EUT: St. Petersburg Paradox

Bernoulli's idea has been employed by John von Neumann and Oskar Morgenstern in their *Theory of Games and Economic Behavior* (1944). In the appendix to the second edition (1947) they develop axiomatic foundations for a theory for decision-making under risk.

EUT: Terminology

It is common terminology to refer to risk if the probability distribution is known, and to refer to uncertainty (or ambiguity) if probabilities are not known. This distinction is due to F. Knight (1920).

Knowledge about	referred to as
state	certainty
distribution of states	risk
range of states	uncertainty (ambiguity)
nothing	deep uncertainty

EUT: Axiomatic characterization

The main assumptions:

- The decision-maker (DM) knows the possible states of the world.
- The DM has preferences over the set of possible states of the world.
- The DM knows the probability distribution attached to each choice option.

The latter assumption is relaxed in *Subjective Expected Utility Theory*; see Savage, L.J. (1954) "The Foundations of Statistics".

The preferences must satisfy a set of axioms.

EUT: Axiomatic characterization

Let $\mathcal{G}$ be a set of (simple and compound) gambles $(p_1 \circ g^1, \dots p_k \circ g^k)$. Then a preference ordering on $\mathcal{G}$ should satisfy the following axioms.

Axiom 1

Completeness (COM): For any two gambles g, g' we have $g \succsim g'$ or $g' \succsim g$.

Axiom 2

Transitivity (TRAN): For any three gambles g, g', g'' , if $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.

Axiom 3

Continuity (CON): Suppose, without loss of generality, we label outcomes such that $a_1 \succsim \cdots \succsim a_n$. For any gamble $g \in \mathcal{G}$ there is some probability α such that $g \sim (\alpha \circ a_1, (1-\alpha) \circ a_n)$

EUT: Axiomatic characterization

Axiom 4

Monotonicity (MON): For all probabilities $\alpha, \beta \in [0,1]$,
$(\alpha \circ a_1, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1, (1-\beta) \circ a_n)$ *if and only if* $\alpha \geq \beta$.

Axiom 5

Substitution (SUB): If $g = (p_1 \circ g^1, \ldots p_k \circ g^k)$ *and*
$h = (p_1 \circ h^1, \ldots p_k \circ h^k)$ *and if* $g^i \sim h^i$ *for every* i*, then* $g \sim h$.

Axiom 6

Reduction (RED): Every compound gamble induces a unique simple gamble $g \in \mathcal{G}_s$*. For any gamble* $h \in \mathcal{G}$*, if* g *is induced by* h*, then* $g \sim h$.

EUT: Axiomatic characterization

Theorem 1

(von Neumann and Morgenstern 1947, Marschak 1950, Nash 1950): Let preferences over gambles satisfy axioms COM, TRAN, CON, MON, SUB and RED, then there exists a utility function $u : \mathcal{G} \to \mathbb{R}$ *that represents the ordering* $\succsim$ *and has the expected utility property, i.e.*

$$u(g) = \sum_i p_i u(a_i).$$

Oskar Morgenstern and John von Neumann (1946)

von Neumann-Morgenstern Utilities

Von Neuman-Morgenstern Utilities are cardinal utilities.

Ratios of utility differences are meaningful, i.e., unique for given preferences and given any three outcomes $a \succ b \succ c$. Let α be the (unique) probability such that $b \sim (\alpha \circ a, (1 - \alpha \circ c)$, then

$$\frac{u(a)-u(b)}{u(b)-u(c)} = \frac{1-\alpha}{\alpha}$$

If u represents "$\succsim$", then for arbitrary numbers λ and $\mu > 0$ the utility function $v = \mu u + \lambda$ represents the same preferences.

Risk attitudes

Assume outcomes of a gamble g are levels of wealth w. Expected wealth is given by $E(g) = \sum_i p_i w_i$.
The DM is

- risk averse if $u(E(g)) > u(g)$,
- risk neutral if $u(E(g)) = u(g)$
- risk loving if $u(E(g)) < u(g)$

Consider a risk averse DM. There is a level of wealth w^c, called *certainty equivalent*, such that $u(w^c) = u(g)$. Risk aversion implies $u(E(g)) > u(g)$. We define the *risk premium* as the difference $P = E(g) - w^c$.

Arrow-Pratt measure of risk aversion

We introduce a notion of "more risk averse than" to compare preferences of different DMs (Pratt, Econometrica 1964; Arrow, 1970).

R_a is the coefficient of absolute risk aversion.

$$R_a = -\frac{u''(w)}{u'(w)}.$$

If $\frac{dR_a(w)}{dw} < 0$ the DM's preferences satisfy the DARA-property (DARA: Decreasing Absolute Risk Aversion).

R_r is the coefficient of relative risk aversion.

$$R_r = -w\frac{u''(w)}{u'(w)}.$$

If $\frac{dR_r(w)}{dw} = 0$ the DM's preferences satisfy the CRRA-property (CRRA: Constant Relative Risk Aversion).

Increasing risk

A definition of increasing risk (Rothschild and Stiglitz, JET 1970) Assume a probability distribution $f(x)$ over outcomes x with expected outcome μ. Consider a distribution $g(x)$ that is derived from $f(x)$ by shifting probability mass from the centre to the tails. Going from $f(x)$ to $g(x)$ is called a mean preserving spread.

Theorem 2

Rothschild and Stiglitz (JET, 1970): Consider distributions $f(x)$ and $g(x)$ as described above, then every risk averter prefers $f(x)$ over $g(x)$.

If there is more "noise" in g than in f, then g is a riskier choice.

Back to your choices

$g_1 = 1000$ for sure

or

$$g_2 = \begin{cases} \text{with} & 0.1 \text{ chance} & 5000 \\ \text{with} & 0.89 \text{ chance} & 1000 \\ \text{with} & 0.01 \text{ chance} & 0 \end{cases}$$

If you go for g_1, then EU theory requires that you choose g_4.
If you go for g_2, then EU theory requires that you choose g_3.

$$g_3 = \begin{cases} \text{with} & 0.1 \text{ chance} & 5000 \\ \text{with} & 0.9 \text{ chance} & 0 \end{cases}$$

or

$$g_4 = \begin{cases} \text{with} & 0.11 \text{ chance} & 1000 \\ \text{with} & 0.89 \text{ chance} & 0 \end{cases}$$

But many people go for the sure thing. This is "Allais' paradox".

Subjective Expected Utility Theory

Ellsberg's paradox

Consider an urn with 30 red balls and 60 balls that are either black or yellow.

Choose between:

$g_1 = 100$ if you draw a red ball

or

$g_2 = 100$ if you draw a black ball

Now choose between:

$g_3 = 100$ if you draw a red or a yellow ball

or

$g_4 = 100$ if you draw a black or a yellow ball

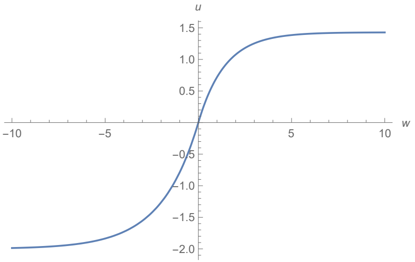
Subjective Expected Utility Theory

If $g_1 \succ g_2$, then your beliefs must be such that $p_{red} > p_{black}$.
But having that belief you should prefer g_3 over g_4 because p_{yellow} is the same for the two gambles.

But many people go for the option with the known probability.
This is "Ellsberg's paradox".

Loss aversion, reference dependent preferences

Daniel Kahneman and Amos Tversky (1979): In many situations people do not evaluate outcomes independent of a ‘status quo’.



The utility function is convex in the domain of losses and concave in the domain of gains. It is also steeper in the domain of losses than in the domain of gains.

Framing

Amos Tversky and Daniel Kahneman (1981) The Framing of Decisions and the Psychology of Choice. Science 211, 453-458.

Problem 1 [N = 152]: Imagine that the U.S. is preparing for the outbreak of an unusual disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows: If Program A is adopted, 200 people will be saved. If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

Which of the two programs would you favor? [Response: 72 percent vs. 28 percent]

Problem2 [N= 155]: If Program C is adopted 400 people will die. If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Which of the two programs would you favor? [Response: 22 percent vs. 78 percent]

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Schoemaker, P.J.H. (1982) The expected utility model: its variants, purposes, evidence and limitations. Journal of Economic Literature 20, 529-563.

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