Advanced Microeconomics(UEC 51806)

Assignment Part 3 (September, October 2024) --- With answers

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Instructions: *Prepare an accurately typed answer sheet. Use a formula editor if you use Word or use Overleaf (overleaf.com – WUR has a campus license). You may work in pairs, i.e., two people can submit joint work. Please submit your work as a pdf-file to Brightspace (BS).*

Upload answers on the BS page before Friday, 18 October 2024, 23.59h

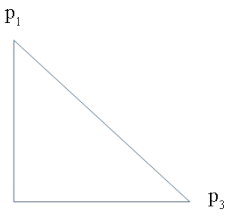
Expected Utility Theory

1. EUT

Axiom 3 (Continuity) asserts that there exists an indifference probability for an gamble in $\mathcal{G}$. Prove for a given gamble $g \in \mathcal{G}$ this probability is unique by using Axiom 4 (Monotonicity).

2. von Neumann-Morgenstern preferences

Consider three outcomes $a_1 \succ a_2 \succ a_3$ occurring with probabilities p_1, p_2, p_3 and a vNM decision-maker. Show that the indifference curves for gambles $(p_1 \circ a_1, p_2 \circ a_2, p_3 \circ a_3)$ are straight lines in the triangle with p_3 and p_1 as axes (see figure).



3. Risk aversion (1)

Climate change policies are designed to control the emissions of greenhouse gases. Suppose, for simplicity, there are only three possible states of the world, either the mean global temperature rises moderately by 1°C, or strongly by 3°C, or very strongly by 5°C.

Consider two policy options: Policy A implies some efforts to abate greenhouse gasses. Policy B is business as usual. The table below gives the probabilities for each possible state of the world and each policy.

	1°C	3°C	5°C
Policy A	0.6	0.3	0.1
Policy B	0.2	0.5	0.3

Suppose, furthermore, that damages due to global warming D are dependent on temperature change ΔT as follows:

$D = 0.5\ (\Delta T)^2\ Y/100$,
where Y is the global income.

a) Calculate the expected temperature change for each policy option.
b) Calculate the expected damage for each policy option.
c) Explain/interpret your findings in a) and b) in words.
d) Greenhouse gas abatement is costly. Assume risk neutrality. How much should society spend to adopt policy A instead of B?
e) Now assume society is risk averse and with a constant relative risk aversion $R_r = \frac{1}{2}$. Assume that risk preferences are described by Atkinson preferences $u(W) = \frac{W^{1-\varepsilon}}{1-\varepsilon}$, W is the income net of damage and abatement cost (measured in GDP) as before. Should policy A or B be adopted?

4. Risk aversion (2)

Find a paper that reports empirical estimates of coefficients of risk aversion. Provide the bibliographical data and briefly report the method and the finding.

Value of Information

5. VOI and risk aversion

Anne can order a box of fresh mushrooms for €15 or not. There may or may not be demand for mushrooms from the local restaurant. If there is demand, then Anne will sell for €50. If there is no demand the mushrooms will rot and cannot be sold. The probability that there is *no demand* is p.

	no demand	demand
not order	0	0
order	-15	35

a) Provide the formula for the expected payoff and calculate the threshold probability where if Anne's prior belief is at this threshold, then *order* and *not order* have the same expected payoffs.
b) Assume that Anne is risk neutral. What would she choose to do?
c) Calculate the value of perfect information about whether or not there is demand. Show that the value of perfect information is a triangular function of Anne's prior belief.
d) Now assume that Anne is risk averse, i.e., her utility function is concave in money. Provide the formula for the expected utility of both actions and solve for the threshold probability.
e) On the basis of this result, argue whether the threshold probability for a risk averse agent is smaller, equal or larger than for the risk neutral agent.

6. Optimal production with price risks

A firm can produce a quantity x of some product for a spot market. Costs are

$$C(x) = x^2,$$

Suppose it can get price p in the market such that revenues are $R = px$.

(a) Calculate the profit maximizing production.

(b) Now suppose the market conditions for the product are unknown but the price is known to be either low p_l or high p_h with probabilities α and $1-\alpha$, respectively. How much should the firm produce.

(c) Consider case (b) and assume $p_l = 2$, $p_h = 10$ and $\alpha = 3/4$. Calculate the optimal production volume.

(d) Using the specification in (c), what is the firm's willingness to pay for a market research that provides perfect information about future prices?

(e) If there is a flat tax on profits, would the firm be willing to pay more or less or an equal amount for the market research when facing a such tax. Explain your answer.

(f) Calculate the value of information if the tax rate is 25%.

(g) Suppose now that the firm faces a progressive tax on profits (such that the average tax increases with profits). Argue qualitatively how this would affect the firm's willingness to pay for information compared to a flat tax rate.

7. The Trust Game (Berg, Dickhaut, McCabe (1995) Games and Economic Behavior)

The Trust game is played as follows. There are two players. Player 1 must decide how much of her show-up fee of \$10 she sends to an anonymous player 2 in another room. Both players are informed that any amount send by player 1 would tripled when it reaches player 2. Of the money player 2 receives he decides to send back some amount.

(i) Assume perfect information and rational and selfish players
   (a) Write down the strategy spaces for both players formally. Draw a game tree.
   (b) Determine the Nash equilibrium of the game. Explain how you find it.

(ii) Now consider incomplete information. There are two types of players. Type *R* is rational and selfish; type *T* is trustworthy and fair. A type *T* player in the role of player 2 would send back ½ of any amount received. There is no difference between types if they are in the role of player 1.
   (c) Draw a game tree of the revised game. Introduce appropriate notation to describe player 1's beliefs about the type of player 2.
   (d) Calculate the expected payoff of player 1 and describe her preferred strategy. Show that the preferred strategy depends on her belief. Derive a threshold probability where her behaviour changes.
   (e) How does player 1's strategy change when she is risk averse? Provide the payoff function of player 1 for this case.

8. A principal-agent problem
There are two agents: a risk neutral land owner and a risk averse farmer. Harvest is subject to risk. The risk is impacted be the farmer's effort. The land owner typically cannot observe the farmer's effort. Only harvest is observable.

a) Argue why or why not the land owner should pay a fixed wage to the farmer.

Assume now that effort *e* is either low $e_l = 0$ or high $e_h = 1$. With low effort harvest is either low $x_l = 15$ with probability $\frac{2}{3}$ or high $x_h = 60$ with probability $\frac{1}{3}$. With high effort harvest is low with probability $\frac{1}{3}$ or high with probability $\frac{2}{3}$. Furthermore assume the farmer can earn an off-farm wage $w_0 = 30.25$ with effort $e_0 = 0.5$. The landowner maximises profits (assume the price of the crop is 1). The farmer's utility function is $u(w,e) = \sqrt{w} - e$.

b) Give a formal description of the land owner's maximisation problem. What are the relevant constraints the land owner faces when offering a contract to the farmer? Describe the contract that the land owner offers to the farmer.

c) Will this contract be accepted?