Advanced Microeconomics

Part 3 (b,c)

a. Expected Utility Theory

b. Value of Information

c. Games with incomplete information

- *solution concepts*
- *insurance (adverse selection)*
- *the principal-agent problem (moral hazard)*

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Decision-making under uncertainty
Part b: Value of information

Information reduces risk.

A simple benchmark model (e.g. Olson 1990)*:

Scheme of payoffs from taking safety measures on the use of a chemical substance

	$\tau = 1$	$\tau = 0$
$\alpha = s$	a	b
$\alpha = l$	c	d

With $d > b \geq a > c$

and a prior belief p_0 that the substance is toxic.

* Olson, Lars J. (1990) The Search for a Safe Environment: The Economics of Screening and Regulating Environmental Hazards. Journal of Environmental Economics and Management 19, 1-18.

Generally, the DM maximises $V \equiv \max_{\alpha} \mathrm{E}\left[U(\alpha,\tau)\right]$

Hence – with two actions and two states – the strict policy is preferred if and only if

$$p_0 a + (1 - p_0) b \geq p_0 c + (1 - p_0) d \quad \text{for } p_0 \in [0,1]$$

Value with prior information

$V_0 = \max\left(p_0 a + (1 - p_0) b,\ p_0 c + (1 - p_0) d \right)$

Conditional probabilities $prob\left(t_1 \middle| \tau\right)$ *of seeing a positive* t_1^+ *or a negative* t_1^- *test outcome for given toxicity*

	$\tau = 1$	$\tau = 0$
t_1^+	φ probability of a true positive result “sensitivity”	$1 - \psi$ probability of a false positive result
t_1^-	$1 - \varphi$ probability of a false negative result	ψ probability of a true negative result “specificity”

Posterior beliefs of a substance being toxic (non-toxic) after evidence from test 1

Bayes's Rule

$p_1(\tau\|\ t_1)$ $p_1(\tau\|\ t_i)$	$\tau = 1$	$\tau = 0$
t_1^+	$p_1^+ = \dfrac{p_0\varphi}{p_0\varphi + (1-p_0)(1-\psi)}$	$(1-p_1^+) = \dfrac{(1-p_0)(1-\psi)}{p_0\varphi + (1-p_0)(1-\psi)}$
t_1^-	$p_1^- = \dfrac{p_0(1-\varphi)}{p_0(1-\varphi) + (1-p_0)\psi}$	$(1-p_1^-) = \dfrac{(1-p_0)\psi}{p_0(1-\varphi) + (1-p_0)\psi}$

Reverend Thomas Bayes (1701-1761)

Check out this one: https://www.youtube.com/watch?v=BrK7X_XlGB8

Now, given the test result we need to distinguish two cases.

(a) For $t_1 = t_1^+$ (positive test result) we have

$$V_1^+ \equiv \max\left[p_1^+ a + (1-p_1^+)b,\ p_1^+ c + (1-p_1^+)d \right].$$

(b) *For* $t_1 = t_1^-$ (negative test result) we have

$$V_1^- \equiv \max\left[p_1^- a + (1-p_1^-)b, p_1^- c + (1-p_1^-)d \right]$$

The expected payoff of performing a test is the weighted average of V_1^+ and V_1^-, where the weights are given by the probability of a test being positive or negative. Thus we have

$$V_1 = \Pr_1^+ V_1^+ + \Pr_1^- V_1^-$$

with

$$\Pr_1^+ = p_0 \varphi + \left(1 - p_0\right)\left(1 - \psi\right) \text{ and } \Pr_1^- = p_0\left(1 - \varphi\right) + \left(1 - p_0\right)\psi .$$

The expected value of information (*VOI*) is defined as the difference between the expected value from an optimal decision on the use of *s* with and without additional evidence from test *i*.

$VOI \equiv V_1 - V_0$.

Since testing is usually costly, test if and only if

$$VOI - k_1 \geq 0. \quad \text{(Under risk neutrality.)}$$

Example: Market research for a new product

Action: produce or not produce
States of the World: small market or large market ($m = S, L$)

Table of payoffs

	S	L
$\neg$ produce	$a = 0$	$b = 0$
produce	$c = -2$	$d = 4$

Assume a prior belief $p_0 = \frac{1}{2}$ that the market is small.

Acting on prior believes: “produce”. Then, $V_0 = 1$.

Market research

Probabilities *of seeing a "small" or "large" market*

	S	L
R^S	$\varphi = 0.8$	$1 - \psi = 0$
R^L	$1 - \varphi = 0.2$	$\psi = 1$

Posterior beliefs of a substance being toxic (non-toxic) after evidence from test 1

Bayes's Rule

$p_1\left(m \middle\vert\; R^m\right)$	S	L
R^S	$p_1^S = \dfrac{p_0\varphi}{p_0\varphi + (1-p_0)(1-\psi)}$	$(1-p_1^S)$
R^L	$p_1^L = \dfrac{p_0(1-\varphi)}{p_0(1-\varphi) + (1-p_0)\psi}$	$(1-p_1^L)$

$p_1\left(m \middle\vert\; R^m\right)$	S	L
R^S	$p_1^S = \dfrac{0.4}{0.4+0}$	0
R^L	$p_1^L = \dfrac{0.1}{0.1+0.5}$	$\frac{5}{6}$

Now, given the test result we need to distinguish two cases.

Recall $a = 0; b = 0; c = -2; d = 4$.

If we find (evidence for) a small market R^S

$$V_1^S \equiv \max\left[p_1^S a + (1 - p_1^S) b,\ p_1^S c + (1 - p_1^S) d \right] = \max\left[0,\ -2 + 0\right] = 0.$$

If we find (evidence for) a large market R^L

$$V_1^L \equiv \max\left[p_1^L a + (1 - p_1^L) b, p_1^L c + (1 - p_1^L) d \right] = \max\left[0, \tfrac{1}{6} \cdot (-2) + \tfrac{5}{6} \cdot 4\right] = \tfrac{18}{6} = 3,$$

Prior to market research your probabilities to see evidence for a “small” or “large” market are

$$\Pr^S = p_0 \varphi + (1 - p_0)(1 - \psi) = 0.4 \ \text{ and } \ \Pr^L = p_0 (1 - \varphi) + (1 - p_0) \psi = 0.6.$$

The expected payoff of performing a test is the weighted average of V_1^S and V_1^L, where the weights are given by the probability of a test being positive or negative. Thus we have

$$V_1 = \Pr^S V_1^S + \Pr^L V_1^L = 0.4 \cdot 0 + 0.6 \cdot 3 = 1.8$$

The expected value of information (*VOI*) is defined as the difference between the expected value from an optimal decision on the use of *s* with and without additional evidence from market research

$$VOI \equiv V_1 - V_0 = 1.8 - 1 = 0.8$$

References: A comprehensive textbook

- Hirshleifer, Jack / Riley, John G. (1992) The Analytics of Uncertainty and Information. Cambridge.

Part c : Games with incomplete information

References

Textbook

- Jehle and Reny, sections 7.2.3–7.3.7 and chapter 8.
- (Kreps, David M. (1990) A course in microeconomic theory. Prentice Hall.)

Seminal works

- Akerlof, George (1970) The Market for “Lemons”: Qualitative Uncertainty and the Market Mechanism. Quarterly Journal of Economics 84, 488-500.
- Stiglitz, Joseph E. (1974) Incentives and Risk Sharing in Sharecropping. Review of Economic Studies 41(2), 219-255.
- Spence, Michael (1973) Job Market Signalling. Quarterly Journal of Economics 87(3), 355-374.
- Rothschild, Michael / Stiglitz, Joseph E. (1976) Equilibrium in competitive insurance markets: an essay on the economics of imperfect information. Quarterly Journal of Economics 90(4), 629-649.

Games and Information: Concepts
(for extensive form games)

- Perfect information: Players, their strategy sets, and their payoffs are known for each stage of the game. At each stage and for each history of the game exactly one player has a non-trivial choice. This implies a simple tree structure of the game. Each information set is a singleton. It also implies perfect recall.
- Imperfect information: Players, their strategy sets and payoffs are known for each stage of the game, but not all aspects of past play. There exists at least one information set that is non-singleton. Examples are simultaneous move games.
- Incomplete information: Some information about other players, their payoffs or their strategies are missing.
- Common knowledge (of rationality) assumption (Robert Aumann): Players are rational, they know that they are rational and they know that they know that they are rational and so forth...

Asymmetric Information

Some players possess information that others do not possess.

- hidden information: One or more players cannot observe the characteristic of another player. Examples are (i) the health condition or driving ability of a customer of an insurance company or (ii) the quality of a good a seller offers (market for lemmons).

  → adverse selection

- hidden action: Some player(s), cannot observe what the other player is doing. However outcome/payoff will depend on those unobservable action. Two prominent examples are the Principal-Agent Problem and moral hazard in teams (Holmstrom 1982).

  → moral hazard

Holmstrom, Bengt (1982) Moral hazard in teams. Bell Journal of Economics 13, 324-340.

A principal-agent problem (Kreps 1990, chapter 16)

A sales agent can work hard (high effort) or not (low effort). His disutility of effort is $a_h = 5$ and $a_l = 0$, respectively. Consider the following utility function $u(w,a) = \sqrt{w} - a$. Let's first consider a benchmark case without uncertainty. Assume the agent brings home orders worth 70 if effort is low and 270 if effort is high. Furthermore the agent has a reservation utility of $\bar{u} = 9$.

What is the reservation wage of the agent?

For low effort: $\sqrt{w} - a_l \geq 9 \Rightarrow \sqrt{w} \geq 9 \Rightarrow w \geq 81$.

For high effort: $\sqrt{w} - a_h \geq 9 \Rightarrow \sqrt{w} \geq 14 \Rightarrow w \geq 196$.

What does the principal offer?

The contract offered pays the agent wage 196 + one cent if effort is high and 25 (say) otherwise.

- It is optimal for the agent to accept and work hard.
- Enforceability is not an issue as effort is assumed to be observable and the contract says that the wage is paid conditional on effort.

Now consider unobservable effort

The salesman (agent) brings home orders worth 400, 100 or 0 depending on luck and effort (see table):

The table shows the conditional probabilities of sales of different sizes

	400	**100**	**0**	**expected sales**
high effort	0.6	0.3	0.1	270
low effort	0.1	0.3	0.6	70

What does the principal believe if he sees a large sale?

Suppose his prior belief that the agent exerts high effort is 0.5. Then, seeing a large sale the belief will be adjusted to 6/7.

The principal is assumed to be risk neutral. Effort is not observable, but the size of the sale is.

Optimal contract offered to a risk neutral agent

In this case assume a linear utility function:

$u(w,a) = w - a;\ \bar{u} = 81; a_h = 25; a_l = 0$.

Hence the reservation wage for hard work is $81 + 25 = 106$.

Principal's expected profits are $270 - 106 = 164$.

For low efforts the principal would rather not hire the agent, since 81 > 70.

What contract should the principal offer?

The wage will be dependent on the size of the sale.

$$w = \begin{cases} -164 & \text{for} \quad \text{no sale} \\ -64 & \text{for} \quad \text{small sale} \\ 236 & \text{for} \quad \text{large sale} \end{cases}$$

Agent exerts high effort, earns the reservation wage and bears all risk. Principal earns 164 for sure. You can check that it does not make sense for the agent to accept the contract and exert low effort.

Optimal contract offered to a risk averse agent

In this case assume as before

$u(w,a) = \sqrt{w} - a\,;\ \bar{u} = 9\,; a_h = 5\,; a_l = 0\,.$

Observe the trade-off:

- **The risk neutral principal should bear all the risk,**
- **But a riskless wage offers no incentives for effort.**

Denote, for convenience, the utility equivalent of w by x, such that $x^2 = w$. The principles problem is to offer a contract

$$w = \begin{cases} x_0^2 & \text{for} \quad \text{no sale} \\ x_1^2 & \text{for} \quad \text{small sale} \\ x_2^2 & \text{for} \quad \text{large sale} \end{cases}$$

The principal maximises profits subject to

- The agent must be willing to accept the contract (participation constraint)
- The agent prefers high effort to low effort (incentive constraint)

Formally:

$$\max\ 270 - (0.1 \cdot x_0^2 + 0.3 \cdot x_1^2 + 0.6 \cdot x_2^2)$$

subject to

(PC) $9 \le 0.1 \cdot x_0 + 0.3 \cdot x_1 + 0.6 \cdot x_2 - 5$

(IC) $0.6 \cdot x_0 + 0.3 \cdot x_1 + 0.1 \cdot x_2 \le 0.1 \cdot x_0 + 0.3 \cdot x_1 + 0.6 \cdot x_2 - 5$

This is left to you as an exercise in constrained optimisation.

We find

$$w = \begin{cases} 5.43^2 & \text{for} \quad \text{no sale} \\ 14^2 & \text{for} \quad \text{small sale} \\ 15.43^2 & \text{for} \quad \text{large sale} \end{cases}$$

The expected wage is 204.6 and exceeds the reservation wage.
But notice that the agent bears some (but not all) risk and he is at his reservation utility level.
Expected profits are $270 - 204.5 = 65.4$.

See also : Grossman, Sanford J. / Hart, Oliver D. (1983) An Analysis of the Principal-Agent Problem. Econometrica 51, 7-45.

Hidden information – the market for "lemmons"

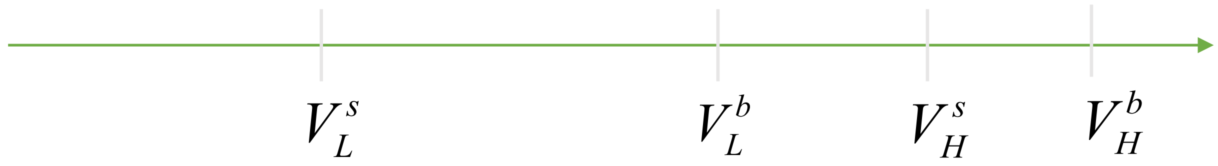
Stylized version: Consider a second hand car market.

Two types of cars of unobservable quality: quality is high or low
sellers' valuation is €2500 or €1000,
buyers' valuation is €3000 or €2000, respectively.
Probability of a "lemon" is 2/3, which is common knowledge.

Fixed and limited supply and infinite demand: a "sellers' market"

If quality is observable:
high quality sells at €3000; low quality sells at €2000
→ no problem!

If quality is not known to anyone:
buyers' valuation is 1/3 €3000 + 2/3 €2000 = €2333.
sellers' valuation is 1/3 €2500 + 2/3 €1000 = €1500.
→ no problem!



But
sellers know the quality and buyers do not!
However price has a signalling function:
Cars offered for less than €2500 must be lemons.
⇒ if the price exceeds €2500 all cars enter the market, but buyers' willingness to pay is only €2333.
⇒ there is no demand for cars if price is above €2000.
Market outcome: only lemons sell at €2000.
This is known as **adverse selection.**

Self-insurance and self-protection (Ehrlich & Becker, JPE 1972)

Ehrlich and Becker study trade-offs between Market insurance, self-insurance and self-protection.

Self-insurance: reduction of loss through own efforts

For low effort: $\max_e : \pi u(w_0 - L(e) - C(e) + (1-\pi)u(w_0 - C(e))$.

Self-protection: reduction of the probability of loss through own efforts

For low effort: $\max_e : \pi(e)u(w_0 - L - C(e) + (1-\pi(e)u(w_0 - C(e))$,

Where $L(e), \pi(e)$ are both decreasing in efforts and $C(e)$ is an increasing effort cost function.