Advanced Microeconomics(UEC 51806)

Assignment Part 3

Lecturer: Hans-Peter Weikard

Instructions: *Prepare an accurately typed answer sheet. Use a formula editor if you use Word or use Overleaf (overleaf.com – WUR has a campus license). You may work in groups of 2-3 people.*

Upload answers on the BS page before Wednesday, 18 October 2023, 14.00h

Expected Utility Theory

1. EUT

On p.100 Jehle and Reny say that AXIOM 4 (Monotonicity) implies $a_1 \succ a_n$. Prove this.

2. Atkinson preferences

In an experiment a person receives an endowment E and a lottery ticket that takes away an amount $0 \le x \le E$ to be invested in a risky project. With probability p the investment is doubled, with probability $1-p$ it is lost. It is implicitly assumed that people have utility functions with the CRRA property (constant relative risk aversion).

Atkinson's utility function is given by $u(w) = \frac{w^{1-\varepsilon}}{1-\varepsilon}$.

a) Show that it has the CRRA property. For which values of the parameter ε is the agent risk averse.
b) Determine the certainty equivalent of the lottery for a general ε.
c) Determine the certainty equivalent for the special case $\varepsilon = 1/2$ and $p = 1/2$.
d) Show for this special case that the certainty equivalent is decreasing in x. Explain why this is to be expected?
e) Is the risk premium de creasing or increasing in *E*? Give an intuitive explanation for your finding.

3. Risk aversion (1)

Anna has expected utility function $a(x)$ and Bert has expected utility function $b(x)$ where x is wealth. Both are risk averse. Let $f : R \to R$ be a monotonically increasing and strictly concave function. Now suppose that

$a(x) = f(b(x))$, that is *a* is an increasing monotonic transformation of *b*.

Show that Anna is more risk averse than Bert in terms of absolute risk aversion. (Hint: be careful when using chain rule to get the second order derivative.)

4. Risk aversion (2)

Find a paper that reports empirical estimates of coefficients of risk aversion. Provide the bibliographical data and briefly report (± 50 words) the method and the finding.

Value of Information

5. Optimal production with price risks

A firm can produce a quantity *x* of some product for a spot market. Costs are

$$C(x) = x^2,$$

Suppose it can get price *p* in the market such that revenues are $R = px$.

(a) Calculate the profit maximizing production.

(b) Now suppose the market conditions for the product are unknown but the price is known to be either low p_l or high p_h with probabilities α and $1-\alpha$, respectively. How much should the firm produce.

(c) Consider case (b) and assume $p_l = 2$, $p_h = 10$ and $\alpha = 3/4$. Calculate the optimal production volume.

(d) Using the specification in (c), what is the firm's willingness to pay for a market research that provides perfect information about future prices?

(e) If there is a flat tax on profits, would the firm be willing to pay more or less or an equal amount for the market research when facing a such tax. Explain your answer.

(f) Calculate the value of information if the tax rate is 25%.

(g) Suppose now that the firm faces a progressive tax on profits (such that the average tax increases with profits). Argue qualitatively how this would affect the firm's willingness to pay for information compared to a flat tax rate.

Games with incomplete information - asymmetric information

7. The Trust Game (Berg, Dickhaut, McCabe (1995) in Games and Economic Behavior)

The Trust game is played as follows. There are two players. Player 1 must decide how much of her show-up fee of $10 she sends to an anonymous player 2 in another room. Both players are informed that any amount send by player 1 would tripled when it reaches player 2. Of the money player 2 receives he decides to send back some amount.

A) Assume perfect information and rational and selfish players
   (a) Write down the strategy spaces for both players formally. Draw a game tree.
   (b) Determine the Nash equilibrium of the game. Explain how you find it.

B) Now consider incomplete information. There are two types of players. Type *R* is rational and selfish; type *T* is trustworthy and fair. A type *T* player in the role of player 2 would send back ½ of any amount received. There is no difference between types if they are in the role of player 1.
   (c) Draw a game tree of the revised game. Introduce appropriate notation to describe player 1's beliefs about the type of player 2.
   (d) Calculate the expected payoff of player 1 and describe her preferred strategy. Show that the preferred strategy depends on her belief. Derive a threshold probability where her behaviour changes.
   (e) How does player 1's strategy change when she is risk averse? Provide the payoff function of player 1 for this case.

8. A principal-agent problem

There are two agents: a risk neutral land owner and a risk averse farmer. Harvest is subject to risk. The risk is impacted be the farmer's effort. The land owner typically cannot observe the farmer's effort. Only harvest is observable.

a) Argue why or why not the land owner should pay a fixed wage to the farmer.

Assume now that effort *e* is either low $e_l = 0$ or high $e_h = 1$. With low effort harvest is either low $x_l = 15$ with probability $\frac{2}{3}$ or high $x_h = 60$ with probability $\frac{1}{3}$. With high effort harvest is low with probability $\frac{1}{3}$ or high with probability $\frac{2}{3}$. Furthermore assume the farmer can earn an off-farm wage $w_0 = 30.25$ with effort $e_0 = 0.5$. The landowner maximises profits (assume the price of the crop is 1). The farmer's utility function is $u(w,e) = \sqrt{w} - e$.

b) Give a formal description of the land owner's maximisation problem. What are the relevant constraints the land owner faces when offering a contract to the farmer? Describe the contract that the land owner offers to the farmer.

c) Will this contract be accepted?