Consider a pure-exchange-economy with consumers A, B and goods $1, 2$. Suppose both consumers have utility function $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$.

1. Give a general result that guarantees that this economy has a walrasian equilibrium (and do not forget to verify the conditions of this result).
2. Determine the walrasian equilibria.
3. Determine an element of the core of this economy.

1. A game in strategic form with two players is called 'strictly competitive' if for all multi-strategies (a, b) and (c, d) it holds that $f^1(a, b) \geq f^1(c, d) \Leftrightarrow f^2(a, b) \leq f^2(c, d)$.

   (a) Show that each zero-sum with two players game is strictly competitive.

   (b) Show that in a strictly competitive game each multi-strategy is strongly pareto efficient.

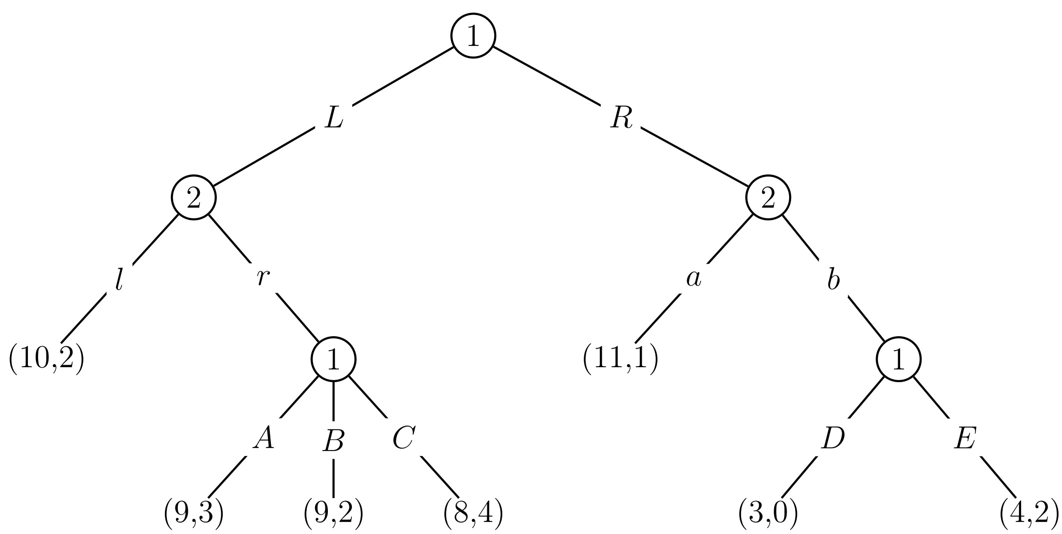
2. Are the following statements about games in strategic form true or false? Prove your answer or give a counter-example.

   (a) If $\mathbf{x}$ is a full cooperative multi-strategy, i.e. a maximiser of the total payoff function, then none of the strategies x^i is strongly dominated.

   (b) In case each player has at least two strategies, there exists a strongly dominated strategy.

   (c) There exists a bi-matrix where no multi-strategy is weakly pareto efficient.

3. Determine the subgame perfect nash equilibria of the following game in extensive form:



A contest game *with complete (part a-c) and incomplete information (d-g).*

There are two agents $i = 1, 2$ competing for a prize P which is worth $v_i(P)$. Both agents invest simultaneously a cost $c_i \in \{1, 2\}$ in order to receive the prize.

Agent i's probability of winning the prize is $\frac{c_i}{c_1 + c_2}$.

a) Determine the payoff function.
b) Draw the game tree that represents the game as an extensive form game.
c) Show that high investments ($c_i = 2$) are a dominant strategy if the valuation of the prize $v_i(P) > 6$.

Now consider incomplete information. Suppose the agents either place a high value on the prize $v_i^H(P) = 9$ or a low value $v_i^L(P) = 3$. The common prior probabilities are given in the following table:

	v_2^L	v_2^H
v_1^L	0.16	0.24
v_1^H	0.24	0.36

d) Draw the game tree that reflects that players' types are randomly determined.
e) Calculate the expected payoff of a player with a high valuation for each of his investment strategies.
f) Describe the Bayesian-Nash equilibria of the game.

A market for micro credit

Consider a loan market to finance investment projects. All projects cost €1. All projects are either good or bad. Only investors know whether their project is good or bad. The bank knows that of all projects a share p are good projects and a share $1-p$ are bad projects. Each project yields either a positive benefit V or zero benefits to the investor. The probability of a positive benefit is higher for good projects. We denote this by $P_g > P_b$. Banks are competitive and risk neutral. A loan contract specifies a repayment R that is repaid to the bank only if the project gives a positive benefit. The opportunity cost of funds (recall project costs are 1) to the bank is $r > 0$. Suppose that

(*) $P_g V - (1+r) > 0 > P_b V - (1+r)$.

a) State what condition (*) says in your own words.
b) Find the equilibrium level of R for a competitive credit market. How does this depend on $P_g, P_b, p, V,$ and r.
c) Discuss which set of projects are financed. Argue that the loan market breaks down completely conditional on some combination of $P_g, P_b, p, V,$ and r.
d) What instrument could the banks use to establish at least a partial market.

Consumer Theory

An infinitely lived agent owns 1 unit of a commodity that he consumes over his lifetime. The commodity is perfectly storable and he will receive no more than he has now. Consumption of the commodity in period *t* is denoted x_t, and his lifetime utility function is given by

$$u(x_0, x_1, x_2, \ldots) = \sum_{t=0}^{\infty} \beta^t \ln x_t \text{, where } 0 < \beta < 1.$$

a.) Write down a general consumer's utility maximization problem. (*1 point*)

b.) Show that the consumer will consume all of his commodity endowment at the optimum. (*1 point*)

c.) Calculate consumer's optimal consumption level in each period. Provide intuition for this consumption pattern.
(*Hint 1*: Feel free to use the Lagrangian function;
Hint 2: Recall from your high school math course that $\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$)
(*3 points*)

d.) What is the marginal utility of having a little bit greater endowment of the commodity at the optimum?
(*1 point*)

e.) Now assume that the consumer will live only *T* years. Calculate consumer's optimal consumption level in each period in this case. How does the consumption pattern compare to your solution in c.)? Explain (*4 points*)

Producer Theory

Provide a brief answer to each of the following questions.

a.) Derive the profit function for the single-output technology whose production function is given by $f(\mathbf{z}) = \sqrt{z_1 + z_2}$. The prices of inputs z_1 and z_2 are w_1 and w_2, respectively. (*3 points*)

b.) Corn (*C*) is produced from labor (*L*) using a decreasing returns to scale technology of the
b.) form $C = AL^{\varepsilon}$, where *A* is a scale parameter and $\varepsilon \in (0,1)$. How is the parameter ε related to the price elasticity of the corn supply curve? (*2 points*)

c.) Ethanol (*E*) is produced from corn (*C*) and labor (*L*) using a Leontief technology

$$E = \min(aC, bL),$$

where *a* and *b* are technological parameters. Draw the inverse ethanol supply curve and determine its price elasticity. (*2 points*)

d.) When the ratio of goods consumed, x_i/x_j, is independent of income (*m*) for all *i* and *j* (i.e., $\partial(x_i/x_j)/\partial m = 0$), then the ratio of any two income elasticities is always equal to 1 (i.e., $\varepsilon_i/\varepsilon_j = 1$). True/false? Show your work. (*3 points*)