Advanced Microeconomics

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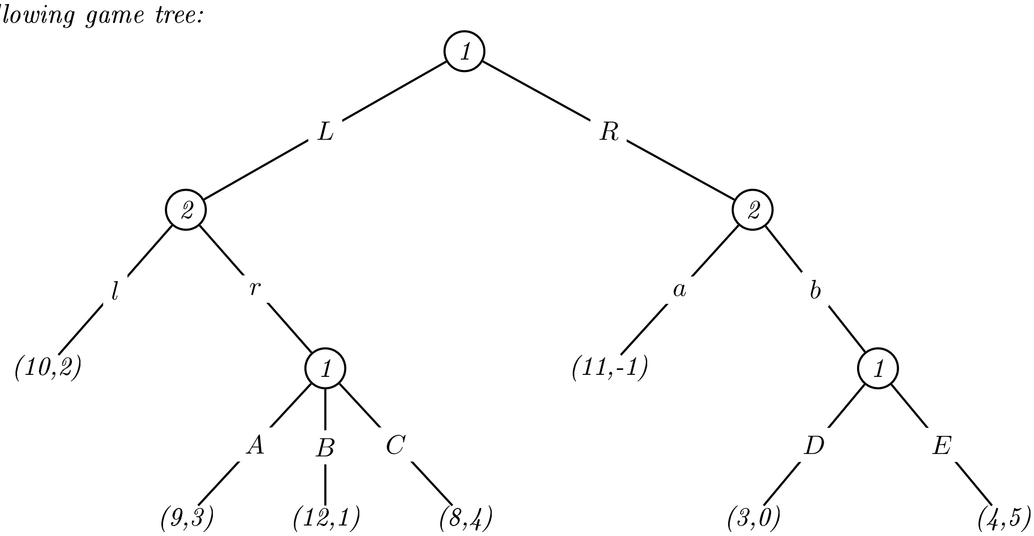
Exercises 4

Remark: Exercises with a * are for next time.

Exercise 1 *Consider the following game between two (rational and intelligent) players. There is a pillow with* 100 *matches. They alternately remove* $1, 2$ *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. Who will win?*

Exercise 2 *Consider the* $2-$*player game in extensive form represented by the following game following game tree:*



a. *How many strategies and which does player* 1 *have? And player* 2*?*

b. ** How may subgames and which does this game have?*

c. *Give a completely elaborated plan of play for player* 1 *that is not a strategy.*

d. ** Determine the subgame perfect equilibria.*

e. *Does this game have a weakly Pareto efficient strategy profile?*

Exercise 3 *Consider an antagonistic game (finite with perfect information). Let* v *be its value and let* (e_1, e_2) *be a Nash equilibrium*

a. *Prove that* $f_1(x_1, e_2) \leq v$ *for each strategy* x_1 *of player* 1 *and* $f_2(e_1, x_2) \leq -v$ *for each strategy* x_2 *of player* 2.

b. *Prove that* e_1 *is a strategy of player* 1 *that guarantees this player at least a payoff* v *and* e_2 *is a strategy of player* 2 *that guarantees this player at least a payoff* $-v$.

Exercise 4 *(The following game is a variant of the so-called ultimatum game.) Player* 2 *says to player* 1 *who has* 10.000 *Euro in his pocket: "Give me that money. If not, then I will detonate the bomb that, You see, I have here with me."*

a. Draw the game tree (with payoffs at the end-notes that are compatible with this situation).

b. Determine for each player the set of strategies.

c. Give the normal form.

d. Determine the strongly dominated strategies.

e. Determine the Nash equilibria.

*f. * Determine the subgame perfect Nash equilibria.*

*g. * How this game probably will be played?*

Short solutions.

Solution 1 By solving the game 'from the end to the beginning' one sees that the loosing positions are those with number of matches that when divided by 3 has remainder 0. As 100 divided by 3 has remainder 1, player 1 will win.

Solution 2 a. Player 1 has 12 strategies: LAD, LAE, LBD, LBE, LCD, LCE, RAD, RAE, RBD, RBE, RCD, RCE. Player 2 has 4 strategies: la, lb, ra, rb.

b. As there are 5 decision nodes, there are 5 subgames. Each game 'hanging below' a decision node is a subgame.

c. I play R, and if I have to make another move, than I play E.

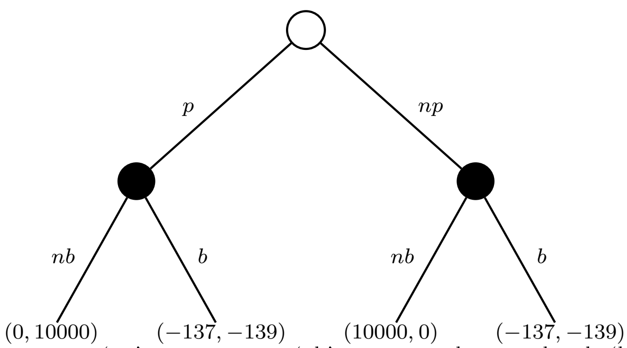
d. Backward induction leads to 1 subgame perfect equilibrium: (LBE, lb).

e. Yes, as the normalisation of this game, being a game in strategic form, is a finite game and each such game has a weakly Pareto efficient strategy profile (even a strongly one).

Solution 3 a. This holds as $v = f_1(e_1, e_2)$ and $f_2 = -f_1$.

b. Because (e_1, e_2) is a Nash equilibrium we have, using part a, for each $x_2 \in X_2$ that $f_1(e_1, x_2) = -f_2(e_1, x_2) \geq v$. In the same way $f_2(x_1, e_2) \geq -v$ for every $x_1 \in X_1$.

Solution 4 a.



Here 'p' means pay, 'np' means no pay, 'nb' means not detonate bomb, 'b' means detonate bomb.

b. Player 1 has 2 strategies: p and np. Player 2 has 4 strategies: at each black node 'nb' (we refer to it by 'always left'), at each black node 'b' (we refer to it by 'always right'), at the left black node 'b' and at the other 'nb' (we refer to it by 'switch'), at the left black node 'nb' and at the other 'b' (we refer to it by 'imitate').

c.

	always left	always right	switch	imitate
p	$0; 10000$	$-137; -139$	$-137; -139$	$0; 10000$
np	$10000; 0$	$-137; -139$	$10000; 0$	$-137; -139$

d. Always right (is strongly dominated by always left).

e. (p,imitate), (np, always left) and (np, switch).

f. The procedure of backward induction gives (np,always left).

g. According to part f: player 1 will not pay and player 2 will not detonate the bomb.