Advanced Microeconomics

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Exercises 1

Exercise 1 *(For this exercise* `http://www.lutanho.net/play/hex.html` *may be useful.)*

a. Play with an opponent two times a hex game: one time You do the first move and in the next game the opponent does.

b. Try to play the game such that it ends in a draw (i.e. no winner, no loser).

Exercise 2 *(For this exercise* `https://playtictactoe.org/` *may be useful.) This exercise deals with the tic-tac-toe game. For parts a and b You need an opponent. For parts b and c use the following standard notation for the cells:*

1	2	3
4	5	6
7	8	9

a. Play the tic-tac-toe game 5 *times.*

b. Both players write a completely elaborated plan of play on a paper that explains how they want to play this game. Then they put both papers on the table and play the game using these strategies.

c. Now suppose You are player 1*. Write a completely elaborated plan of play on a paper that guarantees You at least a draw.*

Exercise 3 *Consider the tic-tac-toe game (with the standard notation of cells) with the following payoff possibilities:* 1 *winning,* 0 *draw,* -1 *loosing.*

a. Show that the following completely elaborated plan of playing player 1 *does not guarantee him at least a draw.*

First move in cell 5*. Each following move according to the first description in the following list that can be applied: (1) Lowest number in same row in which opponent did last move. (2) Lowest number in same column in which opponent did last move. (3) Lowest number.*

b. Convince Yourself that there exists a completely elaborated plan of playing for player 2 *that guarantees him at least draw. (May be by playing the game a lot of times in the position where You are player* 2*.)*

c. Explain in which sense 'draw' is the value of the game.

Exercise 4 *Consider the following Nim-like game. There is a pillow with* m *matches. Both players alternately remove* $1, 2, 3$ *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. Play this game with an opponent for the following values of* m.

a. Suppose $m = 5$.

b. Suppose $m = 10$.

c. Suppose $m = 9$.

d. Suppose $m = 21$.

Exercise 5 *Consider the Hotelling game with parameters m and w (see Lesson 1). Suppose $m = 99$ and an action profile $(24, 24)$. Calculate the payoffs of both players in case*

a. $w = 1$.

b. $w \neq 1$.

Short solutions.

Solution 1 b. This is impossible. (Proving this for an arbitrary board size is very difficult.)

Solution 2 b. For example player 1 uses the following (grim) plan defined by number sequence (317896452), meaning that first move is cell 3 and next move in cell having first number in sequence that is possible for a move.

c. First move: 5. For each next move, if it applies, move opposite to last own move (and then win), otherwise move clockwise beside last move of opponent and if this is not possible, then move anti-clockwise beside last move of opponent.

Solution 3 a. Consider a plan that induces the following moves: $5, 3, 1, 9, 7, 6$. Player 2 wins.

b. You may try to give, as in Exercise 2c, such a plan of playing. However, this may be not so easy. Therefore it is sufficient that You can play the game as player 2 and never loose.

c. Each player can guarantee himself at least a draw. If both players do this, then the result is draw.

Solution 4 Player 2 can win the game in a and b. Player 1 can win the game in c and d.

Solution 5 a. 50.

b.

$$\begin{aligned}
&\Big((w^{24} + w^{23} + \cdots + w^1) + w^0 + (w^{75} + \cdots + w^1)\Big)/2 \\
&= \Big((w^{24} + w^{23} + \cdots + w^1 + w^0) + (w^{75} + \cdots + w^1 + w^0) - w^0\Big)/2 \\
&= \Big(\frac{1 - w^{25}}{1 - w} + \frac{1 - w^{76}}{1 - w} - 1\Big)/2 = \frac{1 + w - w^{25} - w^{76}}{2 - 2w}.
\end{aligned}$$