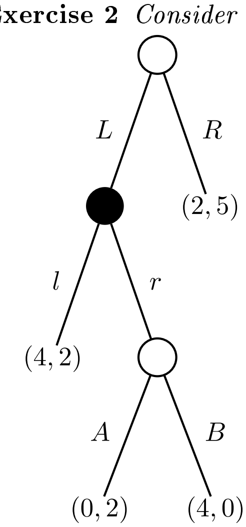
Advanced Microeconomics

P. v. Mouche

Exercises 4

Exercise 1 *Consider the following game between two (rational and intelligent) players. There is a pillow with* 100 *matches. They alternately remove* 1, 2 *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. Who will win?*

Exercise 2 *Consider the following* 2*-player extensive form game given by the game tree*



a. *How many, and which, strategies does player* 1 *have? How many, and which, strategies does player* 2 *have?*

b. *Give a completely elaborated plan of playing for player* 1 *that is not a strategy.*

c. *Determine a normal form for this game.*

d. *Determine for each player the dominant and strictly dominant strategies.*

e. *Determine the Nash equilibria.*

Exercise 3 *(The following game is a variant of the so-called ultimatum game.) Player* 2 *says to player* 1 *who has* 10.000 *Euro in his pocket: "Give me that money. If not, then I will detonate the bomb that, You see, I have here with me."*

a. *Draw the game tree (assuming realistic payoffs).*

b. *Determine for each player the set of strategies.*

c. *Give the normal form.*

d. *Determine the strongly dominated strategies.*

e. *Determine the Nash equilibria.*

f. *Determine the subgame perfect Nash equilibria.*

g. How this game probably will be played?

Exercise 4 *Consider an antagonistic game (finite with perfect information). Let v be its value and let (e_1, e_2) be a Nash equilibrium*

a. Prove that $f_1(x_1, e_2) \leq v$ for each strategy x_1 of player 1 and $f_2(e_1, x_2) \leq -v$ for each strategy x_2 of player 2.

b. Prove that e_1 is a strategy of player 1 that guarantees this player at least a payoff v and e_2 is a strategy of player 2 that guarantees this player at least a payoff $-v$.

Short solutions.

Solution 1 My solving the game 'from the end to the beginning' one sees that the loosing positions are those with number of matches that when divided by 3 has remainder 0. As 100 divided by 3 has remainder 1, player 1 will win.

Solution 2 a. Player 1 has 4 strategies and player 2 has 2 strategies.

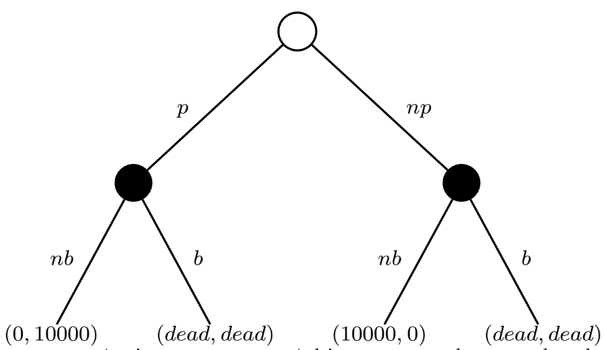
b. Playing R.

c. This is the bimatrix game
$$\begin{pmatrix} & l & r \\ LA & 4;2 & 0;2 \\ LB & 4;2 & 4;0 \\ RA & 2;5 & 2;5 \\ RB & 2;5 & 2;5 \end{pmatrix}.$$

d. Dominant strategies: LB for player 1 and l for player 2. There are no strictly dominant strategies.

e. (LA, l) and (LB, l).

Solution 3 a.



Here 'p' means pay, 'np' means no pay, 'nb' means not detonate bomb, 'b' means detonate bomb.

b. Player 1 has 2 strategies: p and np. Player 2 has 4 strategies: at each black node 'nb' (we refer to it by 'always left'), at each black node 'b' (we refer to it by 'always right'), at the left black node 'b' and at the other 'nb' (we refer to it by 'switch'), at the left black node 'nb' and at the other 'b' (we refer to it by 'imitate').

c.
$$\begin{pmatrix} & \text{always left} & \text{always right} & \text{switch} & \text{imitate} \\ p & 0;10000 & dead;dead & dead;dead & 0;10000 \\ np & 10000;0 & dead;dead & 0;0 & dead;dead \end{pmatrix}$$

d. Always right (is strongly dominated by always left).

e. $(p, imitate)$, $(np, alwaysleft)$ and $(np, switch)$.

f. The procedure of backward induction gives (np,always left).

g. According to part f: player 1 will not pay and player 2 will not detonate the bomb.

Solution 4 a. This holds as $v = f_1(e_1, e_2)$ and $f_2 = -f_1$.

b. Because (e_1, e_2) is a Nash equilibrium we have, using part a, for each $x_2 \in X_2$ that $f_1(e_1, x_2) = -f_2(e_1, x_2) \geq v$. In the same way $f_2(x_1, e_2) \geq -v$ for every $x_1 \in X_1$.