Advanced Microeconomics

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Exercises 2

Exercise 1 *Prove that a fully cooperative strategy profile is strongly Pareto efficient.*

Exercise 2 *Determine which of the following bimatrix games are a prisoner's dilemma.*

a. $\begin{pmatrix} 3;-1 & 3;1 & 6;0 \\ 1;0 & 3;1 & 6;0 \\ 2;2 & 4;1 & 8;2 \end{pmatrix}.$

b. $\begin{pmatrix} 1;0 & 3;1 & 6;0 \\ 2;1 & 4;1 & 8;1 \end{pmatrix}.$

c. $\begin{pmatrix} 6;1 & 3;1 & 1;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}.$

d. $\begin{pmatrix} -1;-1 & 2;0 \\ 0;2 & 3;3 \end{pmatrix}.$

e. $\begin{pmatrix} 2;2 & -1;3 \\ 3;-1 & 0;0 \end{pmatrix}.$

Exercise 3 *Answer the following true/false questions concerning bimatrix games.*

a. A bimatrix game concerns a game with two players.

b. Each bimatrix game has at least one Nash equilibrium.

c. Each bimatrix game has a strictly dominant strategy.

d. Each bimatrix game has a fully cooperative strategy profile.

e. Each bimatrix game has a weakly Pareto efficient strategy profile.

f. Each fully cooperative strategy profile is weakly Pareto efficient.

e. A strictly dominant strategy is fully cooperative.

f. A prisoners' dilemma game has a Nash equilibrium.

g. It is impossible that a weakly Pareto inefficient strategy profile is a Nash equilibrium.

h. A Nash equilibrium is a strategy profile that consists of strategies of the players' that they like the most.

Exercise 4 *The following true/false questions deal with the bimatrix game*

$$\begin{pmatrix} 3;6 & 6;5 & 7;-3 \\ -6;2 & 5;3 & 5;4 \end{pmatrix}.$$

a. *The row-player has* 2 *strategies.*

b. *There are* 6 *strategy profiles.*

c. *The strategy profile* $(1,1)$ *is a Nash equilibrium.*

d. *The row-player has a strictly dominant strategy.*

e. *There is a weakly Pareto inefficient nash equilibrium.*

f. *The column-player has a strictly dominant strategy.*

g. *This game is a prisoners' dilemma.*

h. *Playing row* 1 *and column* 3 *is a fully cooperative strategy profile*

i. *This game is a zero-sum game.*

j. $(1,2)$ *is a weakly Pareto efficient strategy profile.*

Exercise 5 *Consider the Hotelling game in the case* $n = 2$ *(so there are three vertices) and* $w = 1$ *(i.e. inelastic case). Determine the Nash equilibria of this game*

a. *Represent this game as* 3×3*-bi-matrix game with at the first row strategy* 0 *for player* 1*, at the second row strategy* 1 *for player* 1*, etc.*

b. *Determine the Nash equilibria, the strongly Pareto efficient strategy profiles and the weakly Pareto efficient strategy profiles.*

Short solutions.

Solution 1 We prove this by contradiction. So suppose $\mathbf{x}$ is fully cooperative and $\mathbf{x}$ would not be strongly Pareto efficient. Then there exists a pareto improvement $\mathbf{y}$ of $\mathbf{x}$. In $\mathbf{y}$ the sum of payoffs is greater than in $\mathbf{x}$. This is a contradiction with $\mathbf{x}$ being fully cooperative.

Solution 2 Only the game in e is a prisoner's dilemma game.

Solution 3 aT bF cF dT eT fT gF hT iF jF.

Some explanation. Concerning f (each fully cooperative strategy profile is weakly Pareto efficient): suppose the strategy profile $\mathbf{x}$ is fully cooperative, meaning that the total payoff is maximal. If it would not be weakly Pareto efficient, then there is a strategy profile which is better for both players and thus leads to a greater payoff than in $\mathbf{x}$. (In fact each fully cooperative strategy profile even is strongly Pareto efficient. In order to see this modify the above reasoning in an appropriate way.)

Concerning e: as each bimatrix game has a fully cooperative strategy profile, part f implies that each bimatrix game has a weakly Pareto efficient strategy profile.

Solution 4 aT bT cT dT eF fF gF hF iF jT.

Solution 5 a.

$$\begin{pmatrix} 3/2; 3/2 & 1; 2 & 3/2; 3/2 \\ 2; 1 & 3/2; 3/2 & 2; 1 \\ 3/2; 3/2 & 1; 2 & 3/2; 3/2 \end{pmatrix}.$$

b. There is a unique Nash equilibrium: the strategy profile $(1, 1)$.

c. Each strategy profile is strongly Pareto efficient and weakly Pareto efficient (and even fully cooperative).