Seven 25 Euro Problems for the Acid Rain Game

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Abstract

The purpose of this talk is to show that we still do not understand the characteristics of the static ACID RAIN GAME.

The heroes

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- Implicit assumptions due to fact that model is a game in strategic form:
  - Static model with simultaneous strategys.
  - Complete information.
  - Isolated model.

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- The transboundary pollution process is represented by means of a $N \times N$ transport matrix T with elements T_{ij}.The 'portion' $T_{ij}x^j$ of country j's emission level x^j is deposited in country i.This implies that for the emission vector $(x^1, \ldots, x^N)$ the deposition in country j is $Q^j = \sum_{l=1}^{N} T_{jl}x^l$.
- Associated with the deposition Q^j in a country j is a damage cost $\mathcal{D}^j(Q^j)$.

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- Combining the above functions gives the above net benefits function f^j.

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- in the sense that his payoff functions are the sum of damage costs and abatement costs.
- in the sense that his strategy spaces are unbounded (which is more unrealistic).

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4. $\mathcal{D}^j$ *and* θ^j *are continuous;*

5. $\mathcal{D}^j$ *is strictly increasing and convex;*

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6. θ^j is strictly increasing and strictly concave.

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Moreover:

7. *the* $N \times N$*-matrix* $T := (T_{kl})$ *is not diagonal.*

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- We speak of GLOBAL TRANSBOUNDARY POLLUTION if each transport matrix coefficient is 1.

- We call for an emission vector $\mathbf{x} = (x^1, \ldots, x^N)$ the number $Q^j(\mathbf{x}) := \sum_{l=1}^{N} T_{jl} x^l$ the DEPOSITION (LEVEL) in $\mathbf{x}$ of country j.

Easy facts

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- Each ftpg has a unique full cooperative emission vector.

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- smoothness conditions (SMOOTH ftpg) guarantee that one can differential calculus methods.

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IV. The total emission level in the full cooperative emission vector is less than that in a given Nash equilibrium.

V. For each country the deposition level in the full cooperative emission vector is less than that in a given Nash equilibrium.

Note: a ftpg is a prisoners' dilemma game is not a characteristic result, because???

Truth-table

Class / Characteristic	I	II	III	IV	V
Super-smooth, regular and global ftpg	+	+	+	+	+
Super-smooth and regular ftpg	?	+	+	-	?
Super-smooth ftpg	-	?	?	-	?
Regular ftpg	?	?	+	-	?
Global ftpg	-	+	?	+	+
Ftpg	-	?	?	-	?

First line proven by: Chander, Finus, Folmer, v. Mouche, Tulkens.

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Conclusion:

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Conclusion:

May be all characteristics are not valid for the class of ftpgs, that is an ‘everything is possible theorem’ in the style of Sonnenschein-Mantel-Debreu may exist for the class of ftpgs.

Motivation

1. Vermoeden van Birch en Swinnerton-Dyer.
2. Vermoeden van Hodge.
3. Mathematische theorie voor de vergelijkingen van Navier-Stokes.
4. Het P- versus NP-probleem.
5. Het vermoeden van Poincaré.
6. De hypothese van Riemann.
7. Verdere ontwikkeling van de Yang-Mills theorie

Zie http://www.claymath.org voor precieze formuleringen en voor de regels.

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- Identify ftpgs other than those in ??p Theorem 40 where the full cooperative emission vector is a (unanimous) Pareto improvement?? of each Nash equilibrium.
- Is each Nash equilibrium of an ftpg with a transport matrix for which no coefficient is 0 strongly Pareto inefficient?
- Does there exist an ftpg with a Nash equilibrium that is strongly Pareto inefficient but not weakly Pareto inefficient?

References

- This document:
  http://www.amath.washington.edu/~medlock/presentation.html

- K. Mäler, The acid rain game. In: Valuation Methods and Policy Making in Environmental Economics, pages 231–252. Editors: H. Folmer and E. van Ierland. Elsevier, Amsterdam, 1989.

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- P. v. Mouche (2003), Formal transboundary pollution games, in preparation.

- J. Medlock:
  http://www.amath.washington.edu/~medlock/presentation.html

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Child garden of games in strategic form (continued)

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- Prisoners’ dilemma game: a game in strategic form where each player

possesses a strictly dominant strategy such that the strictly dominant equilibrium is weakly Pareto inefficient.

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- A dominant equilibrium is a Nash equilibrium.
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- There are in general no relations between Nash equilibria and full cooperative multi-strategies.
- The Nash equilibrium of a prisoners' dilemma game has a positive social welfare loss, but an ftpg with a unique Nash equilibrium that has a positive social welfare loss is of course not necessarily a prisoners' dilemma.

Hero 2

Hero 2

Hero 3