

Co-operative Game theory - Class room problems

Problem 1 *One interpretation of an egalitarian solution in a cost sharing game with subadditive costs is equal sharing of surplus. Consider a cost sharing game (N, c) with $N = \{1, 2\}$ and the cost (characteristic) function c . The surplus (cost saving) is defined as $W = c(\{1\}) + c(\{2\}) - c(\{1, 2\})$. Then egalitarian surplus sharing means $(x_1, x_2) = (c(1) - \frac{W}{2}, c(2) - \frac{W}{2})$.*

a) *Calculate the egalitarian surplus sharing solution when $c(\{1\}) = 120$; $c(\{2\}) = 140$; $c(\{1, 2\}) = 170$.*

b) *Is this solution in the core? Why?*

Problem 2 *Generalization of egalitarian sharing.*

a) *How can egalitarian surplus sharing be generalized to three players? And to n players?*

b) *Consider now $c(\{1\}) = 120$; $c(\{2\}) = 140$; $c(\{1, 2\}) = 170$, as before and a third player such that $c(\{3\}) = 120$; $c(\{1, 3\}) = 160$; $c(\{2, 3\}) = 190$; $c(\{1, 2, 3\}) = 255$. Calculate the payoffs for egalitarian surplus sharing.*

c) *Show that the solution for d) is not in the core.*

d) *Find the core of the cost sharing game.*

Problem 3 *Consider Problem 1.*

a) *Describe the situation as a bargaining game, i.e. determine the disagreement point and the bargaining set.*

b) *Find the Nash bargaining solution.*

c) *Argue that it must be in the core.*