Advanced Microeconomics: Game Theory

Lesson 5: Games in Extensive Form

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What You will learn

After studying Lesson 5, You

- should understand the notion of subgame perfect Nash equilibrium;
- should know how to perform the procedure of backward induction.

Let us reconsider the game with matches in Lesson 4: there is a pillow with 21 matches. They alternately remove $1, 3$ or 4 matches from it. (Player 1 begins.) The player who makes the last move wins. How this game will be played?

We have shown that in this game player 2 has a winning strategy by analysing the game from the end to the beginning. In fact what there was done is what is called the procedure of backward induction . In fact this procedure applies to each game in extensive form.

Procedure of backward induction

This procedure will be explained by means of the following video which deal with various concrete games.

First see
`https://www.youtube.com/watch?v=pC--lK8KNwo`
(time period 17:07- 19:55).

Next see
`https://www.youtube.com/watch?v=GoeX3fNKghQ`.

Subgame perfect Nash equilibria

So what we have seen in these videos is that in a game in extensive form there may be various Nash equilibria which can be divided in good (i.e. credible) ones and bad (i.e. incredible) ones.

I will explain these things now more systematically.

A credible Nash equilibrium also is called subgame perfect Nash equilibrium . It is a Nash equilibrium that remains a Nash equilibrium in each subgame. Here with subgame we mean the game obtained from the original game tree by starting at one of its decision nodes. (Of course, the original game starts at its initial decision node.)

Subgame perfection

In order to find the subgame perfect Nash equilibria the procedure of backward induction (as explained in the videos) is important.

This procedure (also referred to as ‘pruning the tree’) leads to a non-empty set of strategy profiles, so called backward induction strategy profiles .

Very important results

Here are three important results:

Theorem

(Kuhn.) Each backward induction strategy profile of a finite game in extensive form with perfect information is a Nash equilibrium.

However not each Nash equilibrium is a backward induction strategy profile (i.e. is subgame perfect):

Theorem

For every finite extensive form game with perfect information the set of backward induction strategy profiles coincides with the set of subgame perfect Nash equilibria.

The proofs of both these results are delicate. See text book (if You like).

Very important results (ctd.)

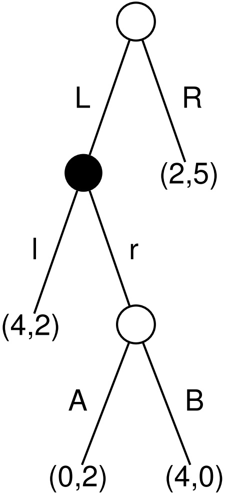
Ad the set of backward induction strategy profiles is not empty, the last result implies:

Theorem

Every finite extensive form game with perfect information has a subgame perfect Nash equilibria.

Example

Let us consider again the following example from Lesson 4:



Example (ctd.

We have seen that player 1 has 4 strategies: *LA*, *LB*, *RA* and *RB*. And that player 2 has 2 strategies: *l* and *r*. This lead to the bimatrix game

$$\begin{pmatrix} & l & r \\ LA & 4;2 & 0;2 \\ LB & 4;2 & 4;0 \\ RA & 2;5 & 2;5 \\ RB & 2;5 & 2;5 \end{pmatrix}.$$

Example (ctd.)

This bimatrix game has two Nash equilibria: $(LA, l), (LB, l)$.

Note that there are 3 subgames, one for each decision node.

Performing the procedure of backward induction we obtain: (LB, l) is a subgame perfect Nash equilibrium, (LA, l) is not a subgame perfect Nash equilibrium.

Outlook

This finishes the theory for the second part of the course. Various extensions of this theory exist; they especially relate to the type of information. In the third part of the course You will study this.

Concerning extensions, i provide now a little outlook (which You may skip if You like).

Three extensions:

- Imperfect information.
- Incomplete information: the solution concept here is that of Bayesian equilibrium (Subsection 7.2.3. in the text book).
- Randomization.

Imperfect information

- Imperfect information can be dealt with by using information sets. The information sets form a partition of the decision nodes. (Example: Figure 7.10 in text book.)
- Perfect information: all information sets are singletons.
- Solution concept: Nash equilibrium.
- Remember: also games in strategic form are games with imperfect information.

Imperfect information (ctd.)

- Strategy: specification at each information set how to move.
- The procedure of backward induction cannot be applied anymore, but the notion of subgame perfect Nash equilibria still makes sense (when 'subgame' is properly defined).
- Subgame: not all decision nodes define anymore a subgame. (Example: Figure 7.20 in text book.)
- Nash equilibria need not always exist. (Example: Figure 7.23 in text book.)

Three types of strategies

Three types of strategies: pure, mixed and behavioural strategies.

- A pure strategy of player i is a book with instructions where there is for each decision node for i a page with the content which move to make at that node. So the set of all pure strategies of player i is a library of such books.
- A mixed strategy of player i is a probability density on his library. Playing a mixed strategy now comes down to choosing a book from this library by using a chance device with the prescribed probability density.

Three types of strategies (ctd.)

- A behavioural strategy, is like a pure strategy also a book, but of a different kind. Each page in the book still refers to a decision node, but now the content is not which move to make but a probability density between the possible moves.
- For many games (for instance those with perfect recall) it makes no difference whatever if players employ mixed or behavioural strategies.

John Nash

- John Nash (1928 – 2015).
- Mathematician. (Economist ?)
- Nobel price for economics in 1994, together with Harsanyi and Selten.
- Abel Price for mathematics in 2015. Just after having received it he was killed in a car crash.
- Got this price for his PhD dissertation (27 pages) in 1950.
- Enjoy looking to the following video about our main hero:

  http://topdocumentaryfilms.com/a-brilliant-madness-john-nash.