Advanced Microeconomics: Game Theory

Lesson 1: Prerequisites

P. v. Mouche

2020, Period 1

Welcome

Welcome to the second part of Advanced Microeconomics!

I'll teach this part, dealing with game theory, in four Lessons.

Lesson 1: Prerequisites.
Lesson 2: Motivation and Outlook.
Lesson 3: Games in Strategic Form.
Lesson 4: Games in Extensive Form

Welcome (ctd.)

This is an advanced course and therefore we expect that You already know various thing. However, for Your convenience, i shall with Lesson 1 from scratch. If You master the prerequisites for the Advanced Micro course, then probably You will not see in this lesson new things or only a few.

Please work through Lesson 1. It is devoted to fundamental game theoretic notions in the context of bimatrix games. In a next lesson i will pick up what is presented here, will generalise and motivate and further elaborate on it.

What You will learn

After studying Lesson 1, You should be able to determine for a bimatrix game

- the dominant strategies;
- the strictly dominant strategies;
- the strictly dominant equilibria;
- the Nash equilibria;
- the strongly Pareto efficient strategy profiles;
- the weakly Pareto efficient strategy profiles;
- the fully cooperative strategy profiles;
- whether the game is a prisoner's dilemma;
- whether the game is a zero-sum game.

These notions are very fundamental; not only in game theory.

Youtube videos

There are a lot of Youtube videos dealing with the above topics. For example: the following video.

`https://www.youtube.com/watch?v=pC--1K8KNwo` (for what we did up to now the video is relevant for period 0:00-7:20 and period 9:05-14:20).

Concerning this video:
1. What is called ‘one-shot game’ in the video we shall call “bimatrix game”.
2. Please forget the terminology ‘normal form game’ in the video (we shall deal later with it).
3. What is called ‘dominant strategy’ in the video, we shall call ‘strictly dominant strategy’.

Bimatrix game

So what is a bimatrix game ? Well, bimatrix games concern the most simple type of so-called games in strategic form dealing with two players; player 1 and player 2. The game is represented by a so-called bimatrix (which explains its name). For example:

$$\begin{pmatrix} 3;3 & 2;2 \\ 7;-1 & -3;1 \\ 1;2 & 12;-9 \end{pmatrix}.$$

- This is a 3×2-bimatrix game, i.e. it has 3 rows and 2 columns.
- Player 1 chooses a row: row 1, row 2 or row 3, meaning that player 1 has 3 strategies. Player 2 chooses a column: column 1 or column 2, meaning that player 2 has 2 strategies. These choices are made simultaneously and independently.

Bimatrix game (ctd.)

- In each of the cells of the bimatrix there is a pair of numbers, separated by a semicolon. These numbers represent the payoffs; the first number concerns player 1 and the second player 2.
- For example: at the strategy profile $(3, 2)$, i.e. row 3 and column 2, player 1 has payoff 12 and player 2 has payoff -9.

Many games can be represented in a natural way as a bimatrix game. For example stone-paper-scissors:

$$\begin{pmatrix} 0;0 & -1;1 & 1;-1 \\ 1;-1 & 0;0 & -1;1 \\ -1;1 & 1;-1 & 0;0 \end{pmatrix}$$

Many games can be represented in a natural way as a bimatrix game. For example stone-paper-scissors:

$$\begin{pmatrix} 0;0 & -1;1 & 1;-1 \\ 1;-1 & 0;0 & -1;1 \\ -1;1 & 1;-1 & 0;0 \end{pmatrix}$$

Indeed: first strategy is stone, second paper and third scissors. If players make the same choice, then it is draw: payoffs 0 for both. If players make a different choice, then there is a winner with payoff 1 and a looser with payoff -1.

Fundamental notions

Knowing what a bimatrix game is, we can now introduce some notions that are useful for making predictions about how such a game can be played. I focus here just on introducing these notions.

- strategy profile : for each player a strategy. For example the strategy profile $(3, 1)$ means: player 1 chooses strategy (i.e. row) 3 and player 2 chooses strategy 1.

Fundamental notions

Knowing what a bimatrix game is, we can now introduce some notions that are useful for making predictions about how such a game can be played. I focus here just on introducing these notions.

- strategy profile : for each player a strategy. For example the strategy profile $(3, 1)$ means: player 1 chooses strategy (i.e. row) 3 and player 2 chooses strategy 1.
- Strictly dominant strategy of a player: the best strategy of that player independently of strategies of the other players.

Fundamental notions

Knowing what a bimatrix game is, we can now introduce some notions that are useful for making predictions about how such a game can be played. I focus here just on introducing these notions.

- strategy profile : for each player a strategy. For example the strategy profile $(3, 1)$ means: player 1 chooses strategy (i.e. row) 3 and player 2 chooses strategy 1.
- Strictly dominant strategy of a player: the best strategy of that player independently of strategies of the other players.
- (Weakly) dominant strategy of a player: a best strategy of that player independently of strategies of the other players.

Fundamental notions

Knowing what a bimatrix game is, we can now introduce some notions that are useful for making predictions about how such a game can be played. I focus here just on introducing these notions.

- strategy profile : for each player a strategy. For example the strategy profile $(3, 1)$ means: player 1 chooses strategy (i.e. row) 3 and player 2 chooses strategy 1.
- Strictly dominant strategy of a player: the best strategy of that player independently of strategies of the other players.
- (Weakly) dominant strategy of a player: a best strategy of that player independently of strategies of the other players.
- Strictly dominant equilibrium : strategy profile in which each strategy is strictly dominant.

Fundamental notions

Knowing what a bimatrix game is, we can now introduce some notions that are useful for making predictions about how such a game can be played. I focus here just on introducing these notions.

- strategy profile : for each player a strategy. For example the strategy profile $(3, 1)$ means: player 1 chooses strategy (i.e. row) 3 and player 2 chooses strategy 1.
- Strictly dominant strategy of a player: the best strategy of that player independently of strategies of the other players.
- (Weakly) dominant strategy of a player: a best strategy of that player independently of strategies of the other players.
- Strictly dominant equilibrium : strategy profile in which each strategy is strictly dominant.
- Nash equilibrium : strategy profile with the property that no player regrets his choice.

Fundamental notions (ctd.)

- A strategy profile **b** is an unanimous Pareto improvement of a strategy profile **a** if each player has in **b** a greater payoff than in **a**.
- A strategy profile **b** is a Pareto improvement of a strategy profile **a** if at least one player has in **b** a greater payoff than in **a** and no player has in **b** a smaller payoff than in **a**.
- A strategy profile **x** is weakly Pareto-efficient if there does not exist an unanimous Pareto-improvement of **x**.
- A strategy profile **x** is weakly Pareto-inefficient if there exists an unanimous Pareto-improvement of **x**.

Fundamental notions (ctd.)

So one also can say: a strategy profile is weakly Pareto efficient if there is no other strategy profile in which each player is better off. And a strategy profile is strongly Pareto efficient if there is no other strategy profile in which at least one player is better off and no player is worse off.

Thus there are two Pareto efficiency notions. Both, of course, are interesting. May be the most important notion in economics is that of Pareto-efficiency. If one speaks about Pareto efficiency one usually means strong Pareto-efficiency. Note that weak Pareto efficiency is a simpler notion than strong Pareto efficiency.

Fundamental notions (ctd.)

A strategy profile is

- fully cooperative if the total payoff in this strategy profile is maximal.

A Prisoners' dilemma game is a game with a strictly dominant equilibrium that is Pareto inefficient.

Finally: a zero-sum game is a game where the total payoff is zero in each strategy profile.

Examples

1. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 2;4 & 1;4 & 4;3 & 3;0 \\ 1;1 & 1;2 & 5;2 & 6;1 \\ 1;2 & 0;5 & 3;4 & 7;3 \\ 0;6 & 0;4 & 3;4 & 1;5 \end{pmatrix}.$$

Examples

1. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 2;4 & 1;4 & 4;3 & 3;0 \\ 1;1 & 1;2 & 5;2 & 6;1 \\ 1;2 & 0;5 & 3;4 & 7;3 \\ 0;6 & 0;4 & 3;4 & 1;5 \end{pmatrix}.$$

   No dominant strategies. No strictly dominant strategies. Nash equilibria: strategy profiles $(1,1), (1,2), (2,2)$ and $(2,3)$.

   Attention: a notation as $(2,3)$ here above denotes the strategy profile where player 1 plays row 2 and player 2 plays column 3. So it deals with strategies and not with payoffs (which in strategy profile $(2,3)$ are 5 for player 1 and 2 for player 2).

Examples (ctd.)

2. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 6;1 & 7;1 & 6;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}$$

Examples (ctd.)

2. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 6;1 & 7;1 & 6;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}$$

Strictly dominant strategies for player 1: strategy 1. Dominant strategy for player 1: strategy 1. Strictly dominant strategies for player 2: none. Dominant strategy for player 2: none. Nash equilibria: strategy profile $(1,3)$.

Examples (ctd.)

3. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 3;-1 & 3;1 & 6;0 \\ 1;0 & 3;1 & 8;0 \\ 5;2 & 4;1 & 8;2 \end{pmatrix}.$$

Examples (ctd.)

3. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 3; -1 & 3; 1 & 6; 0 \\ 1; 0 & 3; 1 & 8; 0 \\ 5; 2 & 4; 1 & 8; 2 \end{pmatrix}.$$

No strictly dominant strategies. Dominant strategy for player 1: strategy 3. Dominant strategy for player 2: none. Nash equilibria: strategy profiles $(3, 1)$ and $(3, 3)$.

Examples (ctd.)

4. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 1;0 & 1;2 & 0;4 \end{pmatrix}.$$

Examples (ctd.)

4. Determine the dominant strategies, the strictly dominant strategies and the Nash equilibria for

$$\begin{pmatrix} 1;0 & 1;2 & 0;4 \end{pmatrix}.$$

Strictly dominant strategies for player 1: strategy 1. Dominant strategies for player 1: strategy 1. Strictly dominant strategy for player 1: strategy 3. Dominant strategy for player 2: strategy 3. Nash equilibria: strategy profiles $(1,3)$.

Examples (ctd.)

5. Determine the strongly and weakly Pareto efficient strategy profiles for

$$\begin{pmatrix} 1;0 & 3;1 & 6;0 \\ 2;1 & 4;1 & 8;1 \end{pmatrix}.$$

Examples (ctd.)

5. Determine the strongly and weakly Pareto efficient strategy profiles for

$$\begin{pmatrix} 1;0 & 3;1 & 6;0 \\ 2;1 & 4;1 & 8;1 \end{pmatrix}.$$

Weakly Pareto efficient strategy profiles: (1,2), (2,1), (2,2), (2,3). Strongly Pareto efficient strategy profiles: (2,3).

Examples (ctd.)

6. Determine the strongly and weakly Pareto efficient strategy profiles for

$$\begin{pmatrix} 6;1 & 3;1 & 1;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}.$$

Examples (ctd.)

6. Determine the strongly and weakly Pareto efficient strategy profiles for

$$\begin{pmatrix} 6;1 & 3;1 & 1;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}.$$

Weakly Pareto efficient strategy profiles:
$(1,1),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)$.
Strongly Pareto efficient strategy profiles:
$(1,1),(1,3),(2,1),(3,2),(3,3)$.

For example: $(2,2)$ is not strongly Pareto efficient as $(3,2)$ is a Pareto improvement of $(2,2)$.

Examples (ctd.)

7. Determine the fully cooperative strategy profiles for

$$\begin{pmatrix} 1;0 & 1;-4 & 0;1 \\ 1;1 & 0;2 & -2;0 \end{pmatrix}.$$

Examples (ctd.)

7. Determine the fully cooperative strategy profiles for

$$\begin{pmatrix} 1;0 & 1;-4 & 0;1 \\ 1;1 & 0;2 & -2;0 \end{pmatrix}.$$

Fully cooperative strategy profiles: $(2,1), (2,2)$.

Examples (ctd.)

8. Determine the strictly dominant equilibria for the following game. Is the game a prisoner's dilemma game?

$$\begin{pmatrix} 1;0 & -1;4 & 0;2 \\ 0;6 & 0;2 & 0;3 \end{pmatrix}.$$

Examples (ctd.)

8. Determine the strictly dominant equilibria for the following game. Is the game a prisoner's dilemma game?

$$\begin{pmatrix} 1;0 & -1;4 & 0;2 \\ 0;6 & 0;2 & 0;3 \end{pmatrix}.$$

No player has a strictly dominant strategy; therefore there is no strictly dominant equilibrium and the game is not a prisoners' dilemma.

Examples (ctd.)

9. Determine the strictly dominant equilibria for the following game. Is the game a prisoner's dilemma game?

$$\begin{pmatrix} -1; -1 & 2; 0 \\ 0; 2 & 3; 3 \end{pmatrix}.$$

Examples (ctd.)

9. Determine the strictly dominant equilibria for the following game. Is the game a prisoner's dilemma game?

$$\begin{pmatrix} -1;-1 & 2;0 \\ 0;2 & 3;3 \end{pmatrix}.$$

Both players have a strictly dominant strategy: their second one. So $(2,2)$ is a strictly dominant equilibrium. As $(2,2)$ is weakly Pareto efficient, the game is not a prisoners' dilemma game.

Examples (ctd.)

10. Determine the dom. and strictly dom. strategies, the strictly dom. equilibria, the Nash eq. the weakly and strongly Pareto eff. strat. profiles and the fully coop. strat. prof. for

$$\begin{pmatrix} -1;0 & -1;1 & 0;0 \\ 2;-2 & -3;3 & -1;3 \\ 4;-3 & 5;-5 & 1;-7 \\ 3;-3 & 3;-5 & -6;8 \end{pmatrix}.$$

Examples (ctd.)

10. Determine the dom. and strictly dom. strategies, the strictly dom. equilibria, the Nash eq. the weakly and strongly Pareto eff. strat. profiles and the fully coop. strat. prof. for

$$\begin{pmatrix} -1;0 & -1;1 & 0;0 \\ 2;-2 & -3;3 & -1;3 \\ 4;-3 & 5;-5 & 1;-7 \\ 3;-3 & 3;-5 & -6;8 \end{pmatrix}.$$

Strictly dom. strategies for player 1: strategy 3. Dom. strategies for player 1: strategy 3. Strictly dom. strategies for player 1: none. Dom. strategies for player 1: none. Strictly dom. equilibria: none. Nash eq. strat. profile $(3, 1)$. Weakly Pareto eff. strat. prof.: $(1, 1), (1, 2), (1, 3), (2, 1),$ $(2, 2), (2, 3), (3, 1), (3, 2), (4, 1), (4, 3)$.

Examples (ctd.)

Strongly Pareto efficient strat. prof.: $(1,3), (2,1), (2,3), (3,1), (3,2), (4,3)$. Fully cooperative strat. prof.: $(2,3), (4,3)$.

For example: $(4,2)$ is not weakly Pareto efficient as $(3,1)$ is an unanimous Pareto improvement of $(4,2)$. And $(1,2)$ is not strongly Pareto efficient as $(2,3)$ is a Pareto improvement of $(1,2)$.

11. Determine the weakly and strongly Pareto efficient strategy profiles for

$$\begin{pmatrix} 3;8 & 4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;4 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

11. Determine the weakly and strongly Pareto efficient strategy profiles for

$$\begin{pmatrix} 3;8 & 4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;4 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

Strongly: (1,2) (2,3). Weakly: (1,1), (1,2) (2,3), (3,2).