Advanced Microeconomics

P. v. Mouche

Exercises 6

The following exercises are taken out of previous exams.

Exercise 1 *The following* 7 *questions concern the bimatrix game*

$$\begin{pmatrix} 5;7 & 6;0 & 3;5 \\ 3;0 & 0;5 & 2;10 \\ -7;6 & 5;7 & 3;-2 \end{pmatrix}.$$

a. Determine the Nash equilibria.

b. Determine the strictly dominant strategies.

c. Determine the weakly and strictly Pareto efficient strategy profiles.

d. Determine the strongly dominated strategies.

e. Is this game a prisoner's dilemma game?

f. Determine for each player his best-reply correspondence.

g. Does player 1 *have a strategy which guarantees him at least* 3 *as payoff?*

Exercise 2 *The following* 9 *statements concern a general bimatrix game. Indicate whether they are true or false.*

a. There are two players and each player has two strategies.

b. If each strategy profile is a Nash equilibrium, then each Nash equilibrium is fully cooperative.

c. There exists a weakly Pareto efficient strategy profile.

d. At least one player has a strongly dominated strategy.

e. A fully cooperative strategy profile can not be a strictly dominant equilibrium.

f. A zero-sum game does not have a Nash equilibrium.

g. Each player has a dictator strategy profile. (A dictator strategy profile of a player is a strategy profile at which this player has a maximal payoff.)

h. A rational player is intelligent.

i. John Nash obtained a Nobel Price for economics.

Exercise 3 *Consider the tic-tac-toe game (with the standard numbering of cells) with the following payoff possibilities:* 1 *winning,* 0 *draw,* -1 *loosing. Show that the following completely elaborated plan of play player* 1 *does not guarantee him at least a draw.*

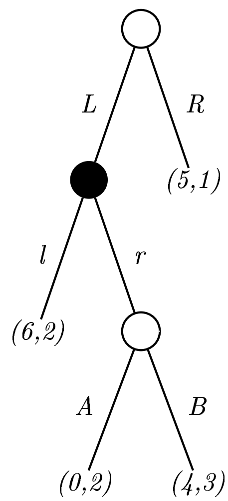
First move in cell 5*. Each following move according to the first description in the following list that can be applied:*
(1) Lowest number in same row in which opponent did last move.
(2) Lowest number in same column in which opponent did last move.
(3) Lowest number.

Exercise 4 *Consider the following game between two players. There is a pillow with* 100 *matches. They alternately remove* $1, 3$ *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. Determine the value of this game.*

Exercise 5 *This exercise deals with a more general case of the Hotelling bimatrix game in Lesson 2. Instead of defining this game here, i refer to the updated slides of this lesson.*

1. *Suppose* n *is even and* $w = 1$*.*
   - a. *Determine the best reply correspondences.*
   - b. *Determine, using the result in 1a, the Nash equilibria.*
   - c. *Show that each strategy profile is strongly pareto-efficient.*
2. *Suppose* $n = 3$ *and* $w = 1/2$*.*
   - a. *Determine the game by representing it as a* 4×4*-bimatrix game with at the first row strategy* 0 *for player* 1*, at the second row strategy* 1 *for player* 1*, etc.*
   - b. *Determine the Nash equilibria and the weakly pareto-efficient strategy profiles.*
   - c. *Determine the strategy profiles that survive the procedure of elimination of strongly dominated strategies.*

Exercise 6 *Consider the following* 2*-player extensive form game with perfect information given by the game tree*



a. *How many, and which, strategies does player* 1 *have? How many, and which, strategies does player* 2 *have?*

b. *Give a completely elaborated plan of play for player* 1 *that is not a strategy.*

c. *Determine a normal form for this game (in terms of a bimatrix).*

d. *Determine all Nash equilibria.*

e. *How many subgames does this game have.*

f. *Determine all subgame perfect Nash equilibria.*

Short solutions.

Solution 1 a. $(1,1)$.
b. No strictly dominant strategies.
c. Weakly: $(1,1),(1,2),(2,3),(3,2)$.
Strongly: $(1,1),(1,2),(2,3),(3,2)$.
d. There are no strongly dominated strategies.
e. No.
f. $R_1(1)=\{1\}, R_1(2)=\{1\}, R_1(3)=\{1,3\}$.
$R_2(1)=\{1\}, R_2(2)=\{3\}, R_2(3)=\{2\}$.
g. Yes, the first strategy.

Solution 2 a.F, b.F, c.T, d.F, e.F., f.F., g.T, h.F, i.T.

Solution 3 Consider the following moves: $5,3,1,9,7,6$. Player 2 wins.

Solution 4 Tthe loosing positions are $0,2,7,9,14,16,21,\ldots$, i.e. the numbers that have remainder 0 or 2 when divided by 7. Because 100/7 has remainder 2, 100 is a loosing position and player 2 has a winning strategy. (So the value is -1.)

Solution 5 1a. As the game is symmetric, the best reply correspondences R_1 and R_2 are the same, just denote it by R. We have $R(x)=\begin{cases}\{x+1\} \text{ if } x<n/2,\\ \{x\} \text{ if } x=n/2,\\ \{x-1\} \text{ if } x>n/2.\end{cases}$

1b. Part 1a implies that there is a unique Nash equilibrium: the strategy profile $(n/2,n/2)$.
1c. As $w=1$ the game is a constant-sum game: $f_1+f_2=n+1$. We have seen in an exercise 1b of week 2 that for such a game each strategy profile is strongly pareto efficient.
2a. For general w the matrix for player 1 is

$$\begin{pmatrix} \frac{1+w+w^2+w^3}{2} & 1 & 1+\frac{w}{2} & 1+w \\ 1+w+w^2 & \frac{1+2w+w^2}{2} & 1+w & 1+w+\frac{w}{2} \\ 1+w+\frac{w}{2} & 1+w & \frac{1+2w+w^2}{2} & 1+w+w^2 \\ 1+w & 1+\frac{w}{2} & 1 & \frac{1+w+w^2+w^3}{2} \end{pmatrix}.$$

For $w=1/2$ the bimatrix becomes

$$\begin{pmatrix} 15/16;15/16 & 1;7/4 & 5/4;7/4 & 3/2;3/2 \\ 7/4;1 & 9/8;9/8 & 3/2;3/2 & 7/4;5/4 \\ 7/4;5/4 & 3/2;3/2 & 9/8;9/8 & 7/4;1 \\ 3/2;3/2 & 5/4;7/4 & 1;7/4 & 15/16,15/16 \end{pmatrix}.$$

2b. Nash equilibria: $(1,2),(2,1)$.
Weakly pareto-efficient strategy profiles: all with exception of $(0,0),(1,1),(2,2)$ and $(3,3)$.
2c. $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$.

Solution 6 a. Player 1 has 4 strategies: LA, RA, LB and RB.
Player 2 has 2 strategies: l and r.
b. Playing R.
c. This is the bimatrix game $\begin{pmatrix} & l & r \\ LA & 6;2 & 0;2 \\ LB & 6;2 & 4;3 \\ RA & 5;1 & 5;1 \\ RB & 5;1 & 5;1 \end{pmatrix}$.
d. $(LA,l),(RA,r),(RB,r)$.
e. 3.
f. (RB,r).