Advanced Microeconomics

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Exercises 5

Exercise 1 *Consider the tic-tac-toe game (with the following cell numbering)*

1	*2*	*3*
4	*5*	*6*
7	*8*	*9*

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a. *Is this a game with perfect information? Is it a game with complete information?*

b. *Explain why this game has a subgame perfect equilibrium. Is there only one subgame perfect equilibrium?*

c. *Show that player* 1 *has at each Nash equilibrium the same payoff. (As usual, the payoffs are:* 1 *for winning,* 0 *for draw and* -1 *for loosing.)*

Exercise 2 *Consider the following game between two players. There is a pillow with* 135 *matches. They alternately remove* 1, 2, 3 *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins.*

a. *True or not true: in the game tree the initial node has* 4 *branches.*

b. *Which player has a winning strategy?*

Exercise 3 *Player* 2 *says to player* 1 *who has* 10.000 *Euro in his pocket: "Give me that money. If not, then I will detonate the bomb that, You see, I have here with me."*

a. *Draw the game tree (assuming realistic payoffs).*

b. *Determine for each player the set of strategies.*

c. *Give the normal form.*

d. *Determine the strongly dominated strategies.*

e. *Determine the Nash equilibria.*

f. *Determine the subgame perfect Nash equilibria.*

g. *How this game probably will be played?*

Short solutions.

Solution 1 1a. Yes.

Yes (each player knows the payoff functions).

1b. As the game is a finite game in extensive form with perfect information it has a subgame perfect equilibrium.

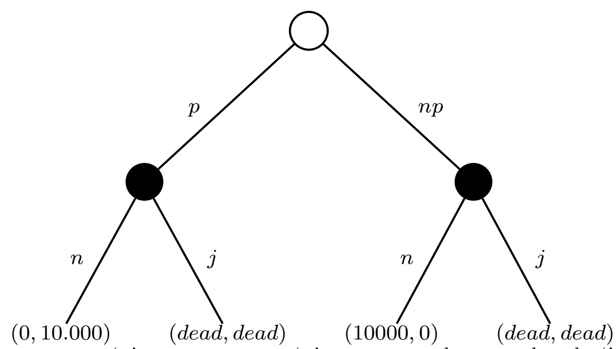
There are may subgame perfect equilibria has the game can be played by both player optimally in many ways.

1c. This holds as the game is an antagonistic game; see Lesson 3.

Solution 2 a. False (as there are a huge amount of ways to take 1, 2, 3 of 4 matches out of the pillow).

b. The losing positions are $0, 5, 10, 15, 20, \ldots$. So player 2 has a winning strategy.

Solution 3 a.



Here 'p' means pay, 'n' means no pay, 'n' means not detonate bomb, 'j' means detonate bomb.

b. Player 1 has 2 strategies: p and np. Player 2 has 4 strategies: at each black node 'n' (we refer to it by 'always left'), at each black node 'j' (we refer to it by 'always right'), at the left black node noop 'j' and at the other 'n' (we refer to it by 'switch'). at the left black node 'n' and at the other 'j' (we refer to it by 'imitate'),

c.

	always left	always right	switch	imitate
p	$0; 10000$	$dead; dead$	$dead; dead$	$0; 10000$
np	$10000; 0$	$dead; dead$	$0; 0$	$dead; dead$

d. Always right (is strongly dominated by always left).

e. (p, imitate), (np, alwaysleft) and (np, switch).

f. The procedure of backward induction gives (np, alwaysleft).

g. According to part f: player 1 will not pay and player 2 will not detonate the bomb.