Advanced Microeconomics

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Exercises 4

Exercise 1 *Consider the tic-tac-toe game (with the standard numbering of cells) with the following payoff possibilities:* 1 *winning,* 0 *draw,* -1 *loosing.*

a. *Show that the following completely elaborated plan of playing player* 1 *does not guarantee him at least a draw.*

   First move in cell 5*. Each following move according to the first description in the following list that can be applied: (1) Lowest number in same row in which opponent did last move. (2) Lowest number in same column in which opponent did last move. (3) Lowest number.*

b. *Show that there exists a completely elaborated plan of playing for player* 1 *that guarantees him at least draw (by giving explicitly such a plan).*

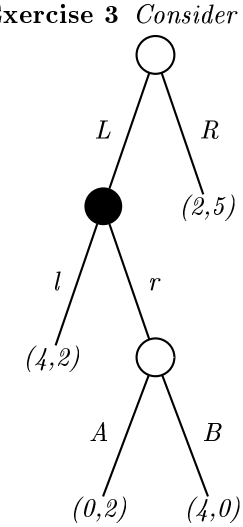
c. *Show directly by a strategy-stealing argument (like for the hex game) that player* 2 *does not have a completely elaborated plan of playing that guarantees winning of the the game.*

d. *Convince Yourself that there exists a completely elaborated plan of playing for player* 2 *that guarantees him at least draw.*

e. *Show that 'draw' is the value of the game.*

Exercise 2 *Consider the following game between two (rational and intelligent) players. There is a pillow with* 100 *matches. They alternately remove* 1, 2 *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. Who will win?*

Exercise 3 *Consider the following* 2*-player extensive form game given by the game tree*



a. *How many, and which, strategies does player* 1 *have? How many, and which, strategies does player* 2 *have?*

b. *Give a completely elaborated plan of playing for player* 1 *that is not a strategy.*

c. Determine a normal form for this game.

d. Determine for each player the dominant and strictly dominant strategies.

e. Determine the Nash equilibria.

Short solutions.

Solution 1 a. Consider the following moves: $5, 3, 1, 9, 7, 6$. Player 2 wins.

b. See Exercise 2b in the exercises belonging to Lesson 2.

c. A strategy-stealing argument for tic-tac-toe goes like this: suppose that the second player has a guaranteed winning strategy, which we will call S. We can convert S into a winning strategy for the first player. The first player should make his first move at random; thereafter he should pretend to be the second player, "stealing" the second player's strategy S, and follow strategy S, which by hypothesis will result in a victory for him. If strategy S calls for him to move in the square that he chose at random for his first move, he should choose at random again. This will not interfere with the execution of S, and this strategy is always at least as good as S since having an extra marked square on the board is never a disadvantage in tic-tac-toe.

d. You may try to give, as in Exercise b, such a plan of playing. However, this may be not so easy. Therefore it is sufficient that You can play the game as player 2 and never loose.

e. By parts b and d.

Solution 2 My solving the game 'from the end to the beginning' one sees that the loosing positions are those with number of matches that when divided by 3 has remainder 0. As 100 divided by 3 has remainder 1, player 1 will win.

Solution 3 a. Player 1 has 4 strategies and player 2 has 2 strategies.

b. Playing R.

c. This is the bimatrix game $\begin{pmatrix} & l & r \\ LA & 4;2 & 0;2 \\ LB & 4;2 & 4;0 \\ RA & 2;5 & 2;5 \\ RB & 2;5 & 2;5 \end{pmatrix}$.

d. Dominant strategies: LB for player 1 and l for player 2. There are no strictly dominant strategies.

e. (LA, l) and (LB, l).