Advanced Microeconomics

P. v. Mouche

Exercises 1

Exercise 1 *Determine which of the following bimatrix games are a prisoner's dilemma.*

a. $\begin{pmatrix} 3;-1 & 3;1 & 6;0 \\ 1;0 & 3;1 & 6;0 \\ 2;2 & 4;1 & 8;2 \end{pmatrix}$.

b. $\begin{pmatrix} 1;0 & 3;1 & 6;0 \\ 2;1 & 4;1 & 8;1 \end{pmatrix}$.

c. $\begin{pmatrix} 6;1 & 3;1 & 1;5 \\ 2;4 & 4;2 & 2;3 \\ 5;1 & 6;1 & 5;2 \end{pmatrix}$.

d. $\begin{pmatrix} -1;-1 & 2;0 \\ 0;2 & 3;3 \end{pmatrix}$.

e. $\begin{pmatrix} 2;2 & -1;3 \\ 3;-1 & 0;0 \end{pmatrix}$.

Exercise 2 *Answer the following true/false questions concerning bimatrix games.*

a. A bimatrix game concerns a game with two players.

b. Each bimatrix game has at least one Nash equilibrium.

c. Each bimatrix game has a strictly dominant strategy.

d. Each bimatrix game has a fully cooperative strategy profile.

e. Each bimatrix game has a weakly Pareto efficient strategy profile.

f. Each fully cooperative strategy profile is weakly Pareto efficient.

e. A strictly dominant strategy is fully cooperative.

f. A prisoners' dilemma game has a Nash equilibrium.

g. It is impossible that a weakly Pareto inefficient strategy profile is a Nash equilibrium.

h. A Nash equilibrium is a strategy profile that consists of strategies of the players' that they like the most.

Exercise 3 *The following true/false questions deal with the bimatrix game*

$$\begin{pmatrix} 3;6 & 6;5 & 7;-3 \\ -6;2 & 5;3 & 5;4 \end{pmatrix}.$$

a. *The row-player has* 2 *strategies.*

b. *There are* 6 *strategy profiles.*

c. *The strategy profile* $(1, 1)$ *is a Nash equilibrium.*

d. *The row-player has a strictly dominant strategy.*

e. *There is a weakly Pareto inefficient nash equilibrium.*

f. *The column-player has a strictly dominant strategy.*

g. *This game is a prisoners' dilemma.*

h. *Playing row* 1 *and column* 3 *is a fully cooperative strategy profile*

i. *This game is a zero-sum game.*

j. $(1, 2)$ *is a weakly Pareto efficient strategy profile.*

Short solutions.

Solution 1 Only the game in e is a prisoner's dilemma game.

Solution 2 aT bF cF dT eT fT gF hT iF jF.

Some explanation. Concerning f (each fully cooperative strategy profile is weakly Pareto efficient): suppose the strategy profile $\mathbf{x}$ is fully cooperative, meaning that the total payoff is maximal. If it would not be weakly Pareto efficient, then there is a strategy profile which is better for both players and thus leads to a greater payoff than in $\mathbf{x}$. (In fact each fully cooperative strategy profile even is strongly Pareto efficient. In order to see this modify the above reasoning in an appropriate way.)

Concerning e: as each bimatrix game has a fully cooperative strategy profile, part f implies that each bimatrix game has a weakly Pareto efficient strategy profile.

Solution 3 aT bT cT dT eF fF gF hF iF jT.